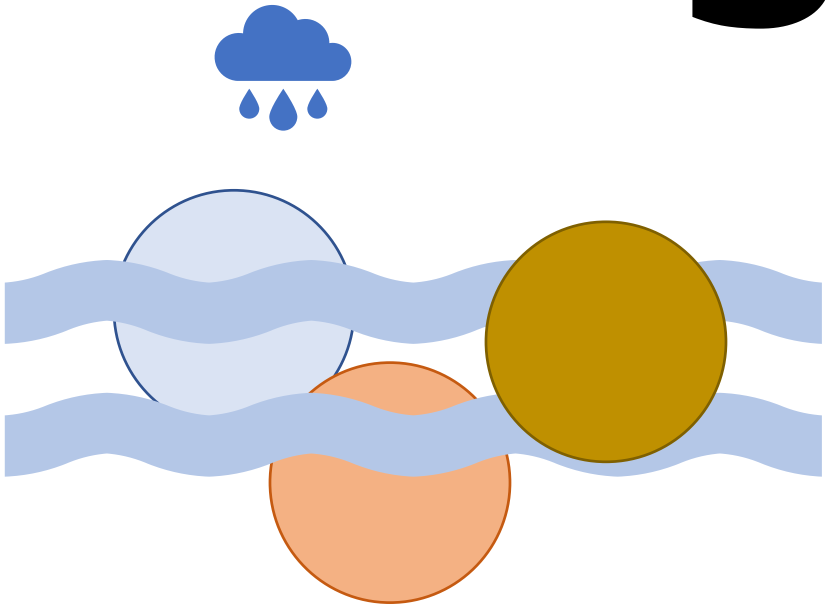
Lake and Reservoir Systems



Standing Waters

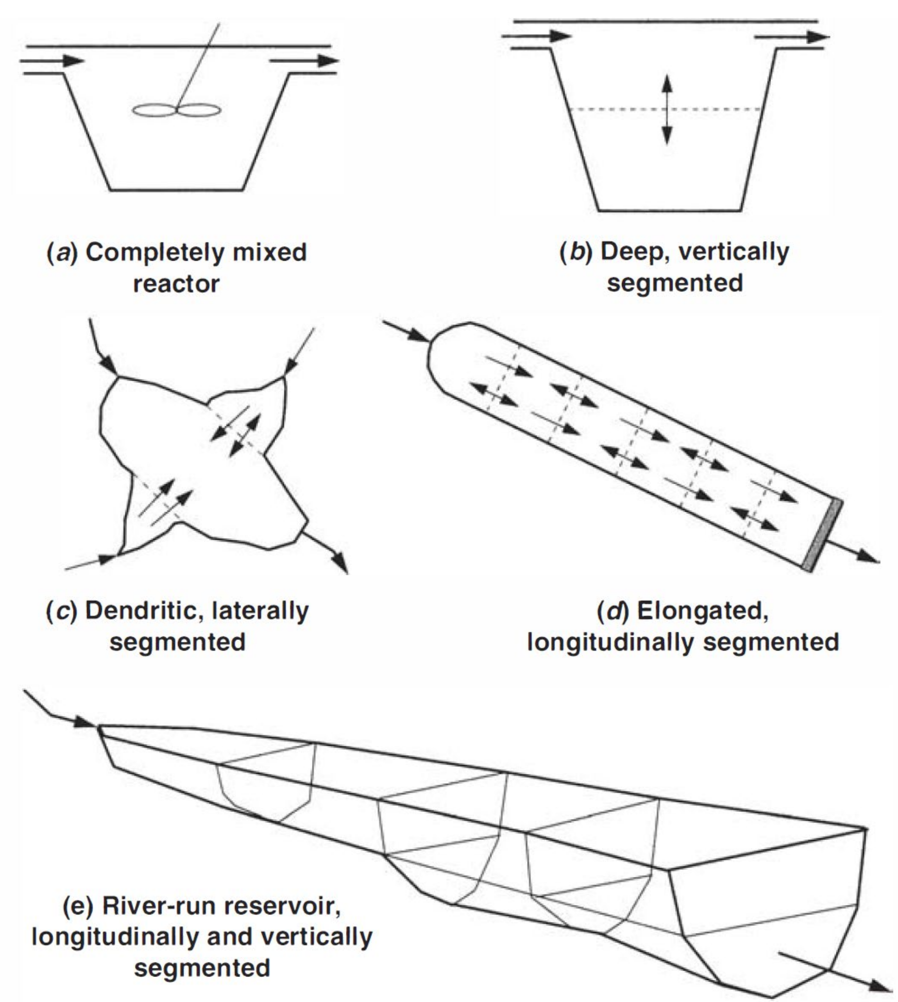


FIGURE 16.1
Typical segmentation schemes used for lakes and impoundments.

Standing water bodies range from small detention ponds to huge systems like the Great Lakes and Lake Baikal. Limnologist (lake scientists) classify lakes in several ways; three major features in particular relate to transport and fate:

Origin: Natural or artificial

Shape: Circular (natural) or elongated/dendritic

Size: residence time (τ_w < 1 yr; short; τ_w > 1yr long)

depth (H< 7m, shallow, H> 7 m,

Lake Morphometry

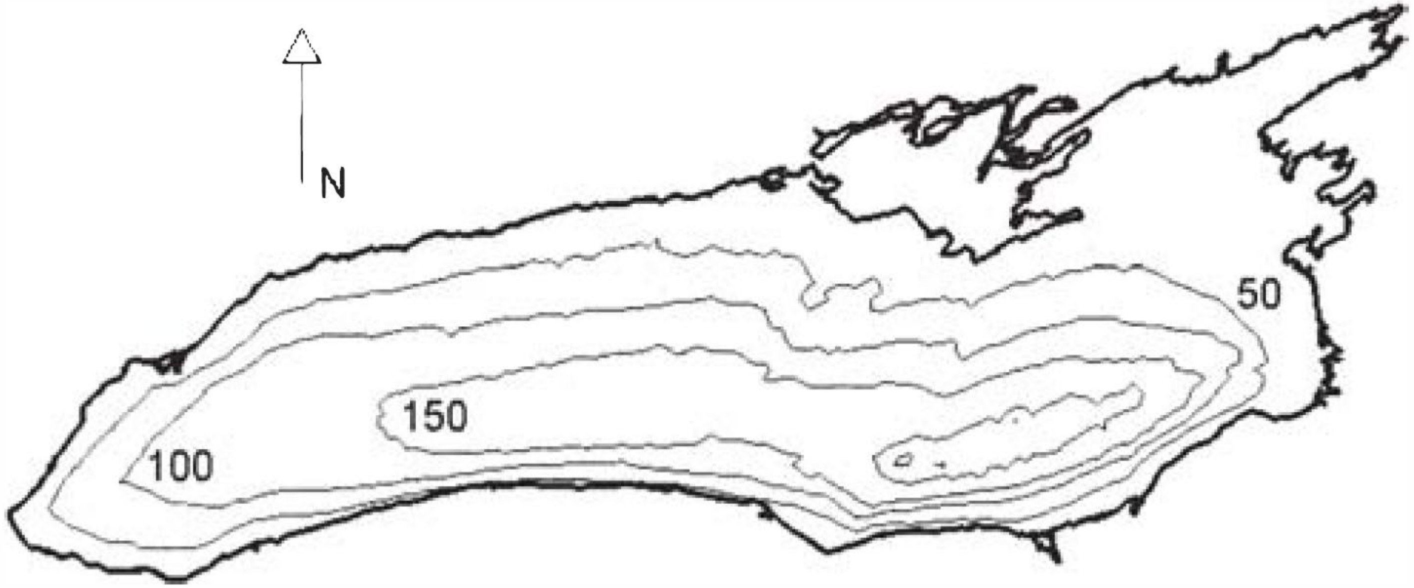


FIGURE 16.2
A bathymetric map of Lake Ontario (courtesy of Mike McCormick, GLERL/NOAA).

Lake Morphometry

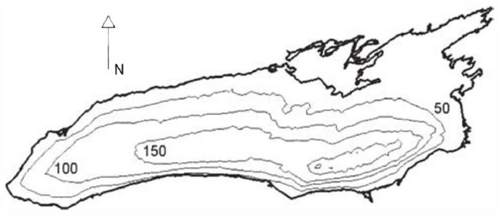


FIGURE 16.2
A bathymetric map of Lake Ontario (courtesy of Mike McCormick, GLERL/NOAA).

The first step is to determine its geometry (formally called its **morphometry**). The **bathymetry** must be mapped (i.e., the topographic map showing depth contour lines). Using a **planimeter**, one can determine how much area is encompassed in each depth contour.

The volume from the surface (z=0) down to a particular depth (z= H) can be calculated as:

$$V(H) = \int_0^H A(z)\,dz$$

Volume between two depths can be evaluated:

$$V_{i,i+1} = \int_{H_i}^{H_{i+1}} A(z)\,dz$$

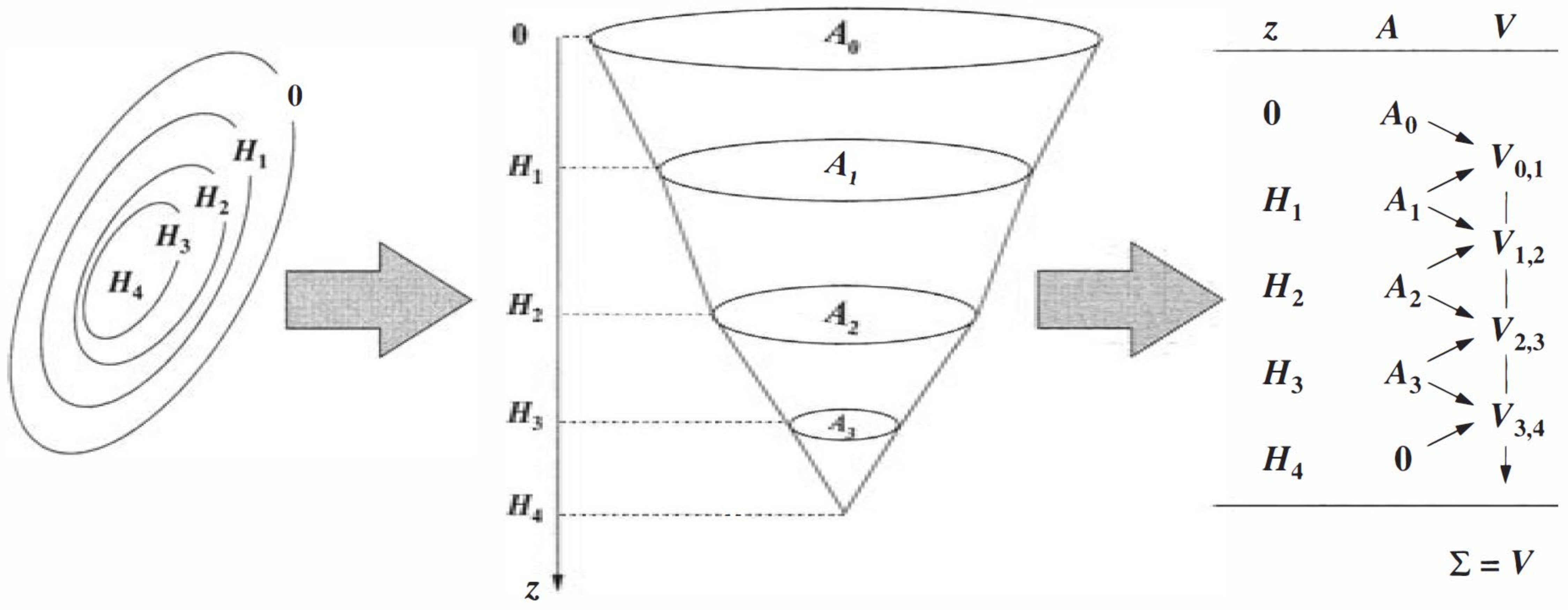
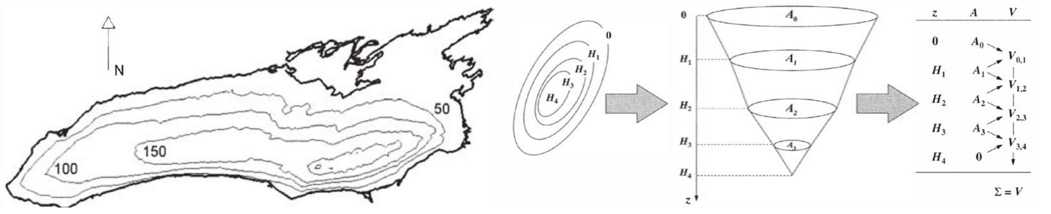


FIGURE 16.3
The process of calculating lake and reservoir morphometry consists of determining areas from a bathymetric map. These areas are then tabulated and used to determine volumes by numerical integration.

Lake Morphometry



Numerical methods must be used to evaluate the integrals, the following is the trapezoidal rule:

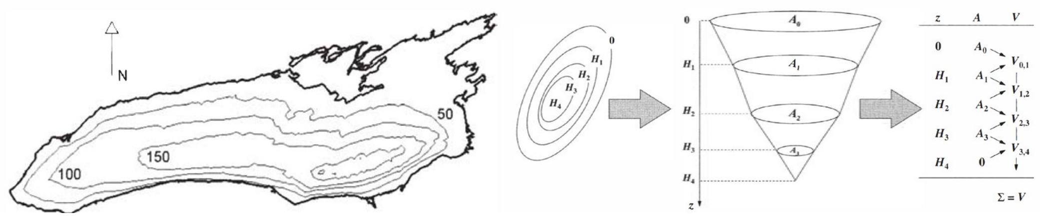
$$V_{i,\,i+1} = \left[\frac{A(H_i) + A(H_{i+1})}{2}\right](H_{i+1} - H_i)$$

Each layer's volume can be determined. Individual layer volumes accumulated to a certain depth:

$$V_{i+1} = \sum_{j=0}^{i}\left[\frac{A\left(H_j\right) + A\left(H_{j+1}\right)}{2}\right](H_{j+1} - H_i)$$

where *i* + 1 = depth at which the volume is to be determined.

Lake Morphometry



The area can also be determined from volumes. Based on the inverse relationship between differentiation and integration.

$$A(H) = \frac{dV(H)}{dz}$$

A numerical approach again is required, below is the centered divided difference:

$$A_i = \frac{dV_i}{dz} \cong \frac{V_{i+1} - V_{i-1}}{z_{i+1} - z_{i-1}}$$

An improvement on this approach would be:

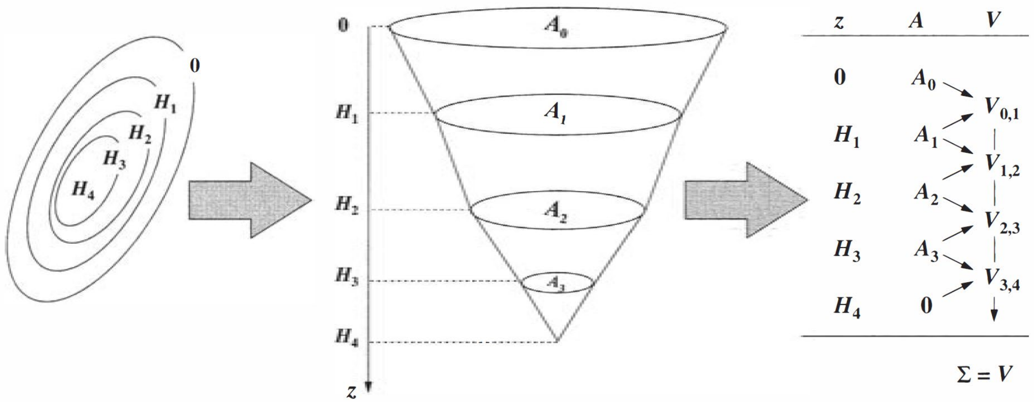
$$A(z) = \frac{dV(z)}{dz}$$

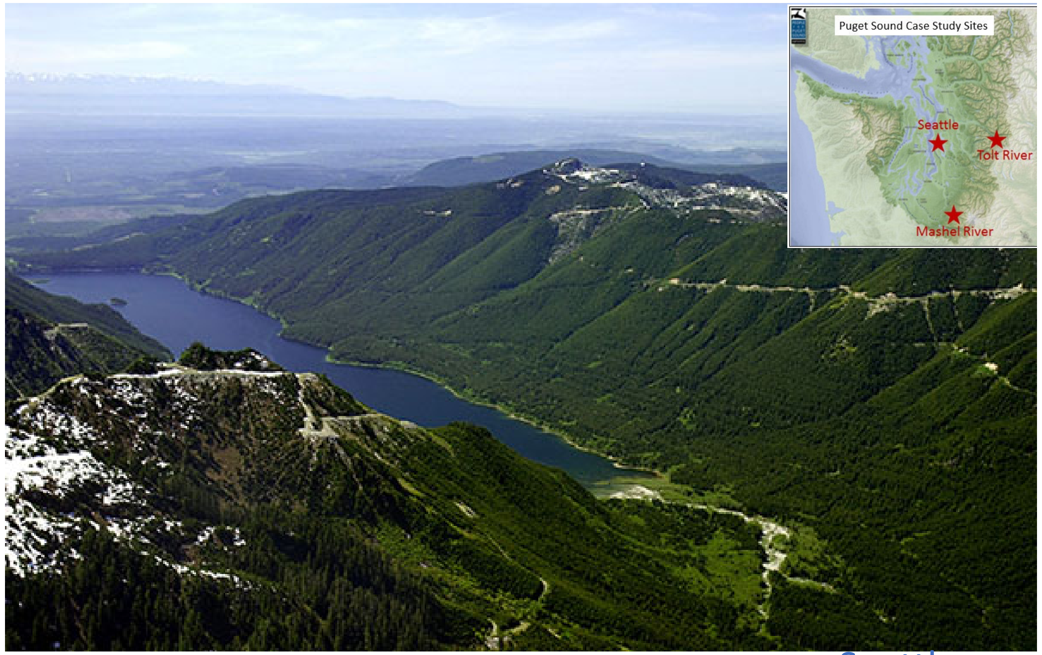
$$\cong V_{i-1}\frac{2z - z_i - z_{i+1}}{(z_{i-1} - z_i)(z_{i-1} - z_{i+1})} + V_i\frac{2z - z_{i-1} - z_{i+1}}{(z_i - z_{i-1})(z_{i-1} - z_{i+1})} + V_{i+1}\frac{2z - z_{i-1} - z_i}{(z_{i+1} - z_{i-1})(z_{i+1} - z_i)}$$

EXAMPLE 16.1. BATHYMETRIC AREA AND VOLUME CALCULATIONS. Bathymetric maps of the Tolt Reservoir (a water-supply impoundment for Seattle, Washington) can be used to determine the areas and depths in the second and third columns of Table 16.1. Determine the volume if the reservoir level is at $z = 0$.

TABLE 16.1
Summary of bathymetric data and morphometric calculations for the Tolt Reservoir, Washington

Index	Depth (m)	Area ($10^6 m^2$)	Volume ($10^6 m^3$)	Cumulative volume ($10^6 m^3$)
0	0	5.180	29.73	0
1	6.10	4.573	13.44	29.73
2	9.14	4.249	12.64	43.17
3	12.19	4.047	11.78	55.81
4	15.24	3.683	10.67	67.59
5	18.29	3.318	16.90	78.26
6	24.38	2.226	11.22	95.16
7	30.48	1.457	7.401	106.4
8	36.57	0.971	4.255	113.8
9	42.67	0.425	1.789	118.0
10	48.77	0.162	0.493	119.8
11	54.86	0	0	120.3





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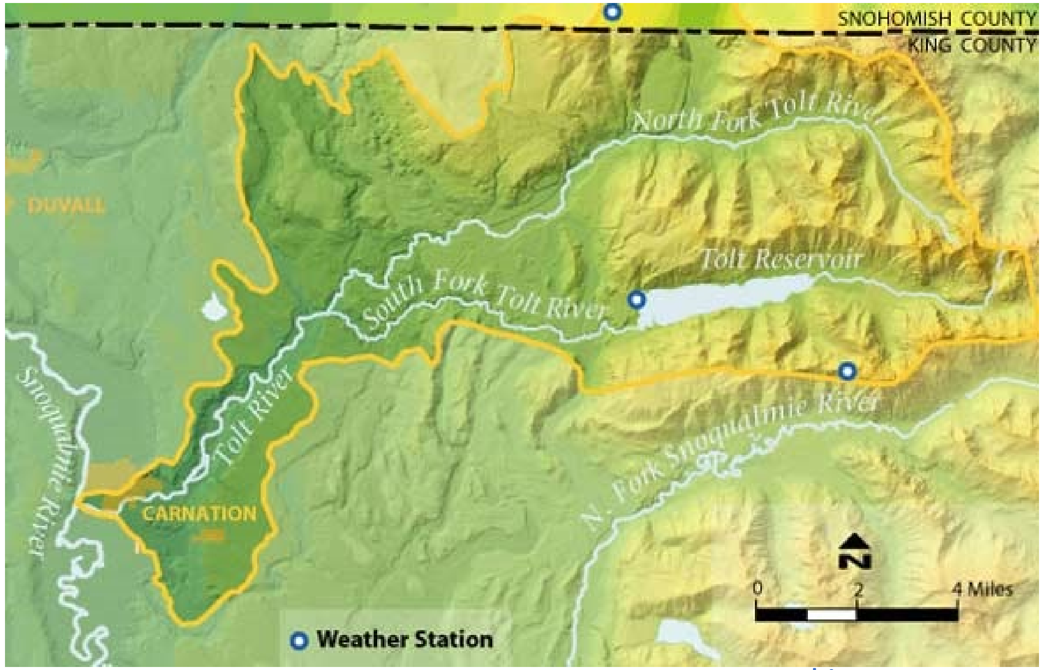

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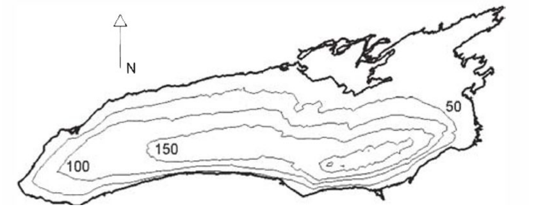
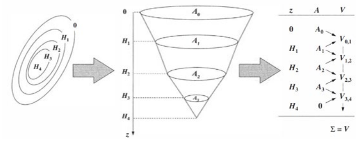
Lake Morphometry



As an example let's test this ($A_i = \frac{dV_i}{dz} \cong \frac{V_{i+1}-V_{i-1}}{z_{i+1}-z_{i-1}}$) as a formula for area at the interface 1.

$$A_1 = \frac{dV_1}{dz} \cong \frac{V_2 - V_0}{z_2 - z_0} = \frac{(43.17 \times 10^6 - 0)}{(9.14 - 0)} = 4.72\ \times 10^6 m^2$$



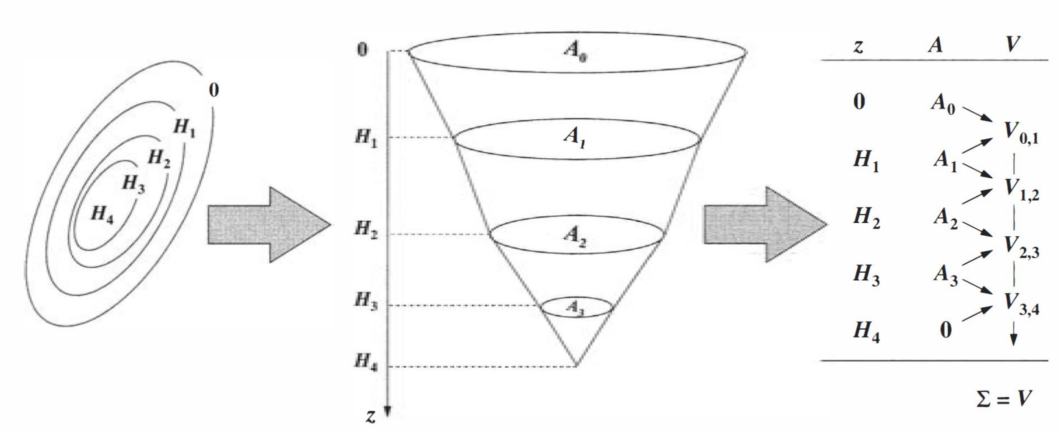
However the true area is $4.573\ \times 10^6 m^2$. With the alternative formula:

$$\frac{dV(6.10)}{dz} \cong 0 + 29.73 \times 10^6 \frac{2(6.10) - 0 - 9.14}{(6.10 - 0)(6.10 - 9.14)}$$
$$+43.17 \times 10^6 \frac{2(6.10) - 0 - 6.10}{(9.14 - 0)(9.14 - 6.10)} = 4.572 \times 10^6 m^2$$

And also at top:

$$\frac{dV(0)}{dz} \cong 0 + 29.73 \times 10^6 \frac{2(0) - 0 - 9.14}{(6.10 - 0)(6.10 - 9.14)}$$
$$+43.17 \times 10^6 \frac{2(0) - 0 - 6.10}{(9.14 - 0)(9.14 - 6.10)} = 5.176 \times 10^6 m^2$$

Rather than numerical differentiation, which tends to amplify data error, instead follow the procedure as follows: Bathymetry → area → volume.



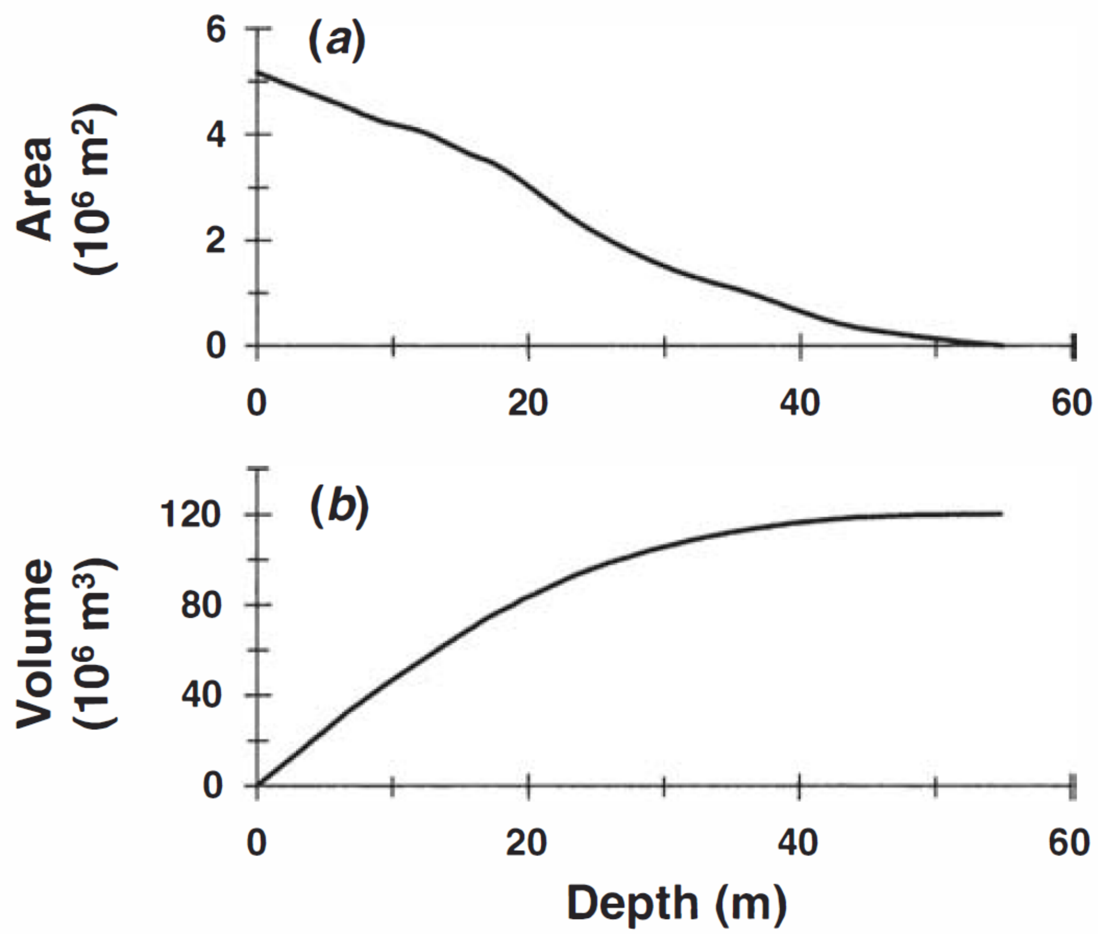


FIGURE 16.4
(*a*) Area-depth and (*b*) volume-depth curves for the Tolt Reservoir, Washington.

Water Balance

A water balance for a well-mixed lake can be written as:

$$S = \frac{dV}{dt} = Q_{in} - Q_{out} + G + PA_s - EA_s$$

where S = storage ($m^3\ d^{-1}$)

V = volume (m^3)

t = time (d)

Q_{in} = inflow ($m^3\ d^{-1}$)

Q_{out} = outflow ($m^3\ d^{-1}$)

G = groundwater flow ($m^3\ d^{-1}$)

P = precipitation ($m\ d^{-1}$)

E = evaporation ($m\ d^{-1}$)

A_s = surface area (m^2)

Seattle.gov

which can be employed for steady-state and time-variable conditions .

WB: Steady-State

For many cases, impoundments (and lakes) tend to not vary drastically in volume for the time periods assessed in WQ-models.

$$S = \frac{dV}{dt} = Q_{in} - Q_{out} + G + PA_s - EA_s$$

simplifies to:

$$0 = Q_{in} - Q_{out} + G + PA_s - EA_s$$

Often inflow and outflow are straightforward measures, however other terms not as available or definable. Sometimes its assumed precipitation balances evaporation and that groundwater flows are negligible (***these assumptions should be validated***).

EXAMPLE 16.2. EFFECT OF WATER BUDGET ON QUALITY MODELING. A lake has the following characteristics:

$$\text{Volume} = 1 \times 10^7 \text{ m}^3$$
$$\text{River inflow} = 1 \times 10^6 \text{ m}^3\text{ d}^{-1}$$
$$\text{River outflow} = 0.8 \times 10^6 \text{ m}^3\text{ d}^{-1}$$

Suppose that a first-order decaying (0.1 d^{-1}) dissolved pollutant is discharged to this system at a constant rate of mass loading of $1 \times 10^7 \text{ g d}^{-1}$. Calculate the lake concentration for two cases, if the discrepancy between inflow and outflow is due to (*a*) a groundwater loss or (*b*) an evaporation loss.

WB: Evaporation

In some cases evaporation can be estimated by difference (balance of inputs, outputs), however in many systems this is impossible. Hence, direct measurements and model equations provide options.

Pan evaporation rate E_p (cm d^{-1}) is one method which is corrected by a pan coefficient k_p to extrapolate to natural waters. These are used to estimate flow through evaporation:

$$Q_e = 0.01 k_p E_p A_s$$

Where Q_e = evaporative water flow (m^{3}d^{-1}), A_s = surface area (m^2), k_p averages as 0.70 for US (between 0.64 to 0.81). 0.01 converts cm to m.

WB: Evaporation

Other methods exists to calculate evaporation based on meterological and lake conditions. Energy flux due to evaporation can be computed as:

$$H_e = f\ (U_w)(e_s - e_{air})$$

where $f(U_w)$ = a function reflecting the effect of wind on evaporation and e_s and e_{air} = the vapor pressure corresponding to the water and the dew-point temperatures (mmHg), respectively. The heat transfer can be converted to a water flow by:

$$Q_e = 0.01\frac{f(U_w)(e_s - e_{air})}{L_e \rho_w} A_s$$

where L_e = the latent heat of vaporization (cal g^{-1}) and ρ_w = water density (g cm^{-3}). The 0.01 allows flow in m^3 d^{-1}.

WB: Evaporation

The latent heat and wind functions can be computed by:

$$L_e = 597.3 - 0.57T$$

And (Brady et al 1969)

$$f(U_w) = 19.0 + 0.95U_w^2$$

where *T*= temperature (°C) and U_w = the wind speed measured in m s^{-1} at a height of 7 m above the water surface. The vapor pressures can be determined by the formula (Raudkivi 1979):

$$e = 4.596e^{\frac{17.27T}{237.3+T}}$$

where the surface water and the dew-point temperature are used to generate e_s and e_{air} respectively. These formulas can be used to calculate E flows and when used in the water balance can also solve for gw.

EXAMPLE 16.3. EVAPORATION CALCULATION. A lake has the following characteristics:

$$\text{Surface area} = 1 \times 10^6 \text{ m}^3$$
$$\text{Wind speed} = 2 \text{ mps}$$
$$\text{Water temperature} = 25^\circ\text{C}$$
$$\text{Dew-point temperature} = 20^\circ\text{C}$$

Compute the evaporation flow.

WB: Time-Variable

For simplicity we assume that precipitation and evaporation are approximately equal, and that groundwater flow is negligible, then the balance becomes:

$$\frac{dV}{dt} = Q_{in} - Q_{out}$$

If the outflow is not known then a relationship must be established between the outflow and head (volume) in the reservoir. For certain spillway structures this is:

$$Q_{out} = CLH^{a}$$

WB: Time-Variable

$$Q_{out} = CLH^a$$

where *C* and *a* = coefficients

L = length of the spillway

H = the total head or surface-water elevation.

when values are not available relationships can be established by measurement of flows and elevation. This then makes the first balance written as:

$$\frac{dV}{dt} = Q_{in}(t) - Q_{out}(H)$$

WB: Time-Variable

$$\frac{dV}{dt} = Q_{in}(t) - Q_{out}(H)$$

Additionally, the relationship of depth and volume is:

$$dV = A(H)dH$$

Thus an area-stage relationship A(H) must be established. If this is done the above equation is substituted into the balance to give:

$$\frac{dH}{dt} = \frac{Q_{in}(t) - Q_{out}(H)}{A(H)}$$

Sometimes called the level-pool routing technique.

EXAMPLE 16.4. LEVEL-POOL RESERVOIR FLOW ROUTING. A small detention pond has a surface area of 2 ha and vertical sides. The discharge-head relationship has been measured:

Elevation (m)	Outflow ($m^3\ s^{-1}$)
0.0	0.0
0.5	0.0
1.0	0.0
1.5	1.7
2.0	5.0
2.5	9.0
3.0	14.0
3.5	20.0
4.0	26.0

Notice that no outflow occurs when the water level is below 1 m. This volume is called ***dead storage***. A storm creates the inflow hydrograph in Fig. 16.5. Use the level-pool model to route this flow through the detention basin. Assume that initially the pond has a depth of 1 m.

Solution: The results of applying the level-pool routing technique are displayed in Fig. 16.5. Notice how the pond tends to diminish and spread the flow. The peak discharge is diminished from 10 cms at $t = 50$ min for the inflow to approximately 4.5 cms at about $t = 78$ min for the outflow.

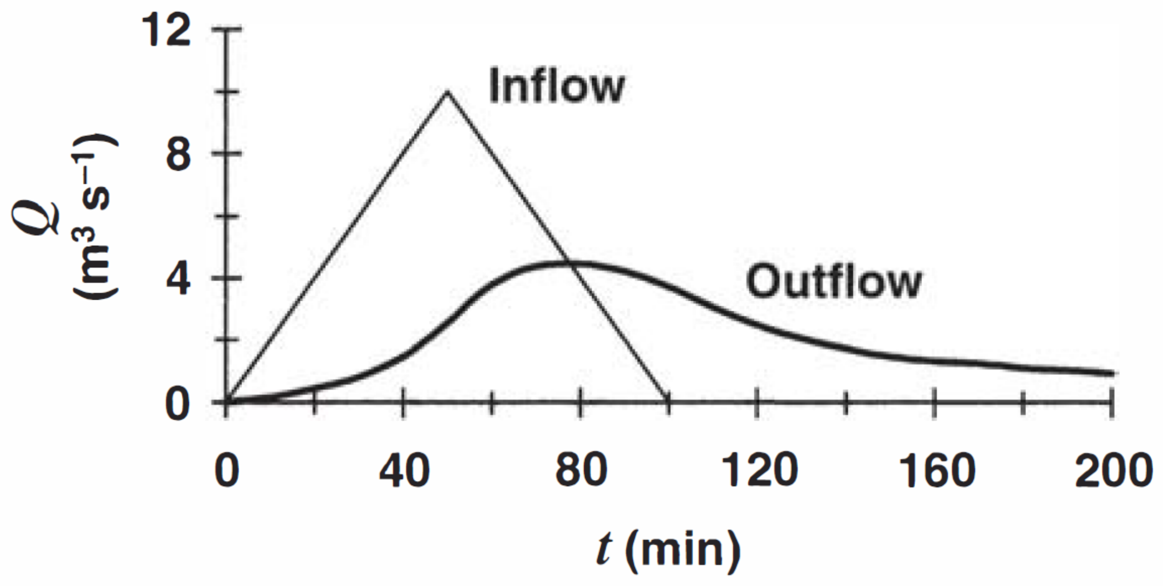


FIGURE 16.5
Inflow and outflow hydrographs for a small detention pond. The outflow hydrograph was calculated with the level-pool routing method.

WB: Time-Variable

To model pollutant transport and fate for a pond, we can write a general mass balance for a pollutant that reacts with first-order kinetics:

$$\frac{dM}{dt} = Q_{in}(t)c_{in}(t) - Q_{out}(H)\frac{M}{V} - kM$$

To determine the ponds concentration we merely use:

$$c = \frac{M}{V}$$

EXAMPLE 16.5. LEVEL-POOL RESERVOIR POLLUTANT ROUTING. Suppose that for the detention pond from Example 16.4, the inflow has a constant concentration of 100 mg L^{-1} of a pollutant that settles at a rate of 1 m d^{-1}. Calculate the concentration and the rate of mass outflow. Assume that $c = 0$ at $t = 0$.

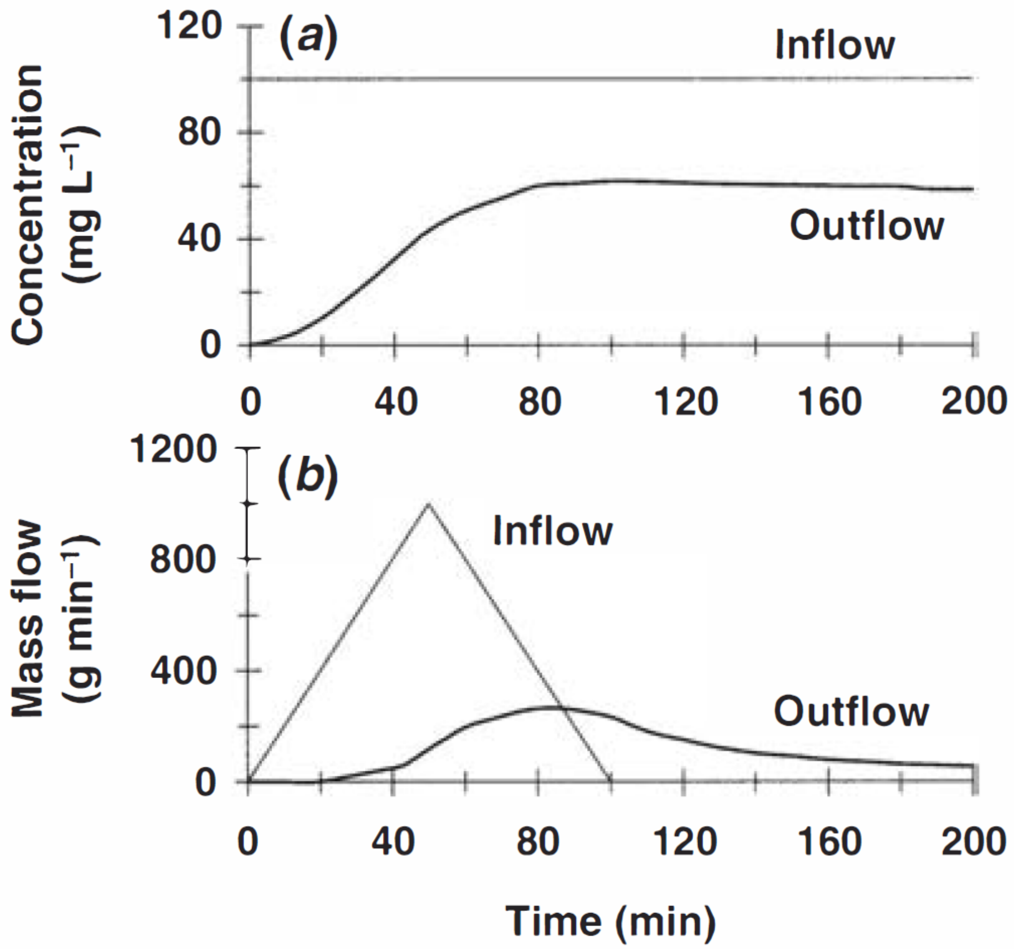


FIGURE 16.6
Inflow and outflow (*a*) concentration and (*b*) mass for a small detention pond.

Near-Shore Model (Advanced Topic)

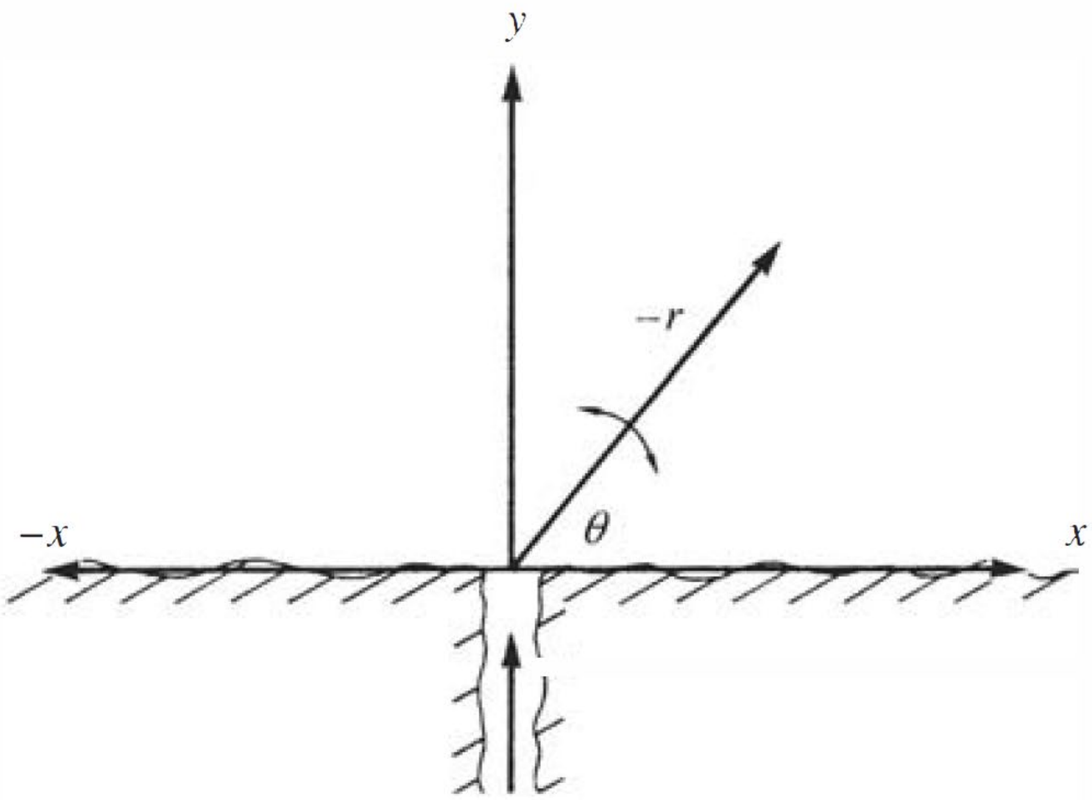


FIGURE 16.7
and radial (r) coordinates used for coastal zone models.

Near-Shore Model (Advanced Topic)

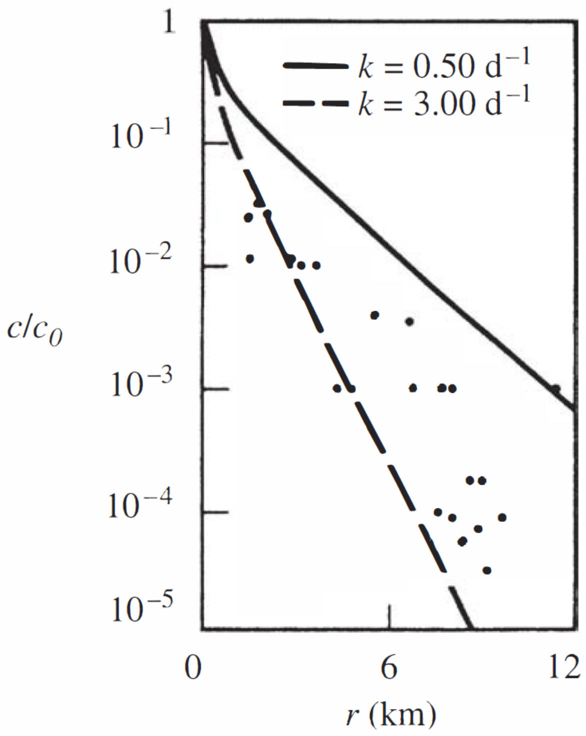


FIGURE 16.8
Profiles of bacterial concentration (normalized to concentration at the edge of the mixing zone) versus distance r (km) from the edge the mixing zone in the vicinity of Indiana Harbor, Lake Michigan, as originally computed by O'Connor (1962). The lines represent model calculations based on two estimates of the bacterial die-off rate. The data are 3-month averages for June through August.

Near-Shore Model (Advanced Topic)

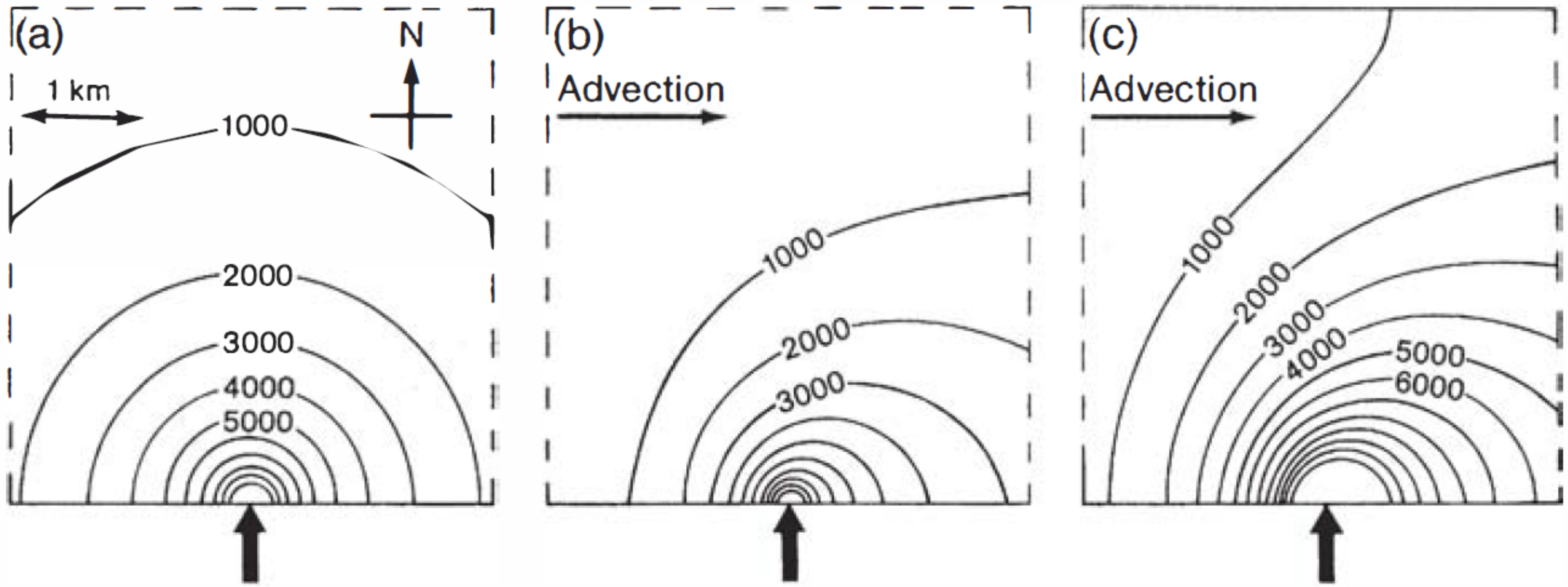


FIGURE 16.9
Contours of coliform bacteria (number per 100 mL) as a result of waste discharge at a point on a lake's shoreline for three cases: (*a*) unbounded fluid with diffusion, (*b*) unbounded fluid with diffusion and advection, and (*c*) bounded fluid with diffusion and advection.