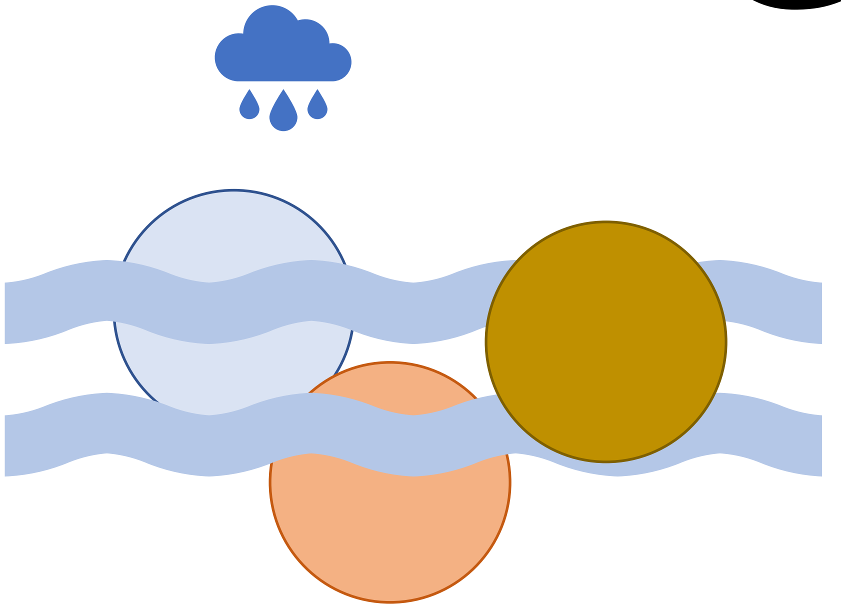
# Phosphorus Loading Concept



**1. Reflection: Have you witnessed a lake's eutrophication ? What indicators would you have measured to check its trophic state?**

**2. Vollenweider Loading Plots**

**3. Budget Models**

**4. Trophic State Correlations: P-Chlorophyll a**

**5. Chlorophyll-Secchi-Disk Depth**

**6. Areal Hypolimnetic Oxygen Demand**

# Vollenweider Loading Plots

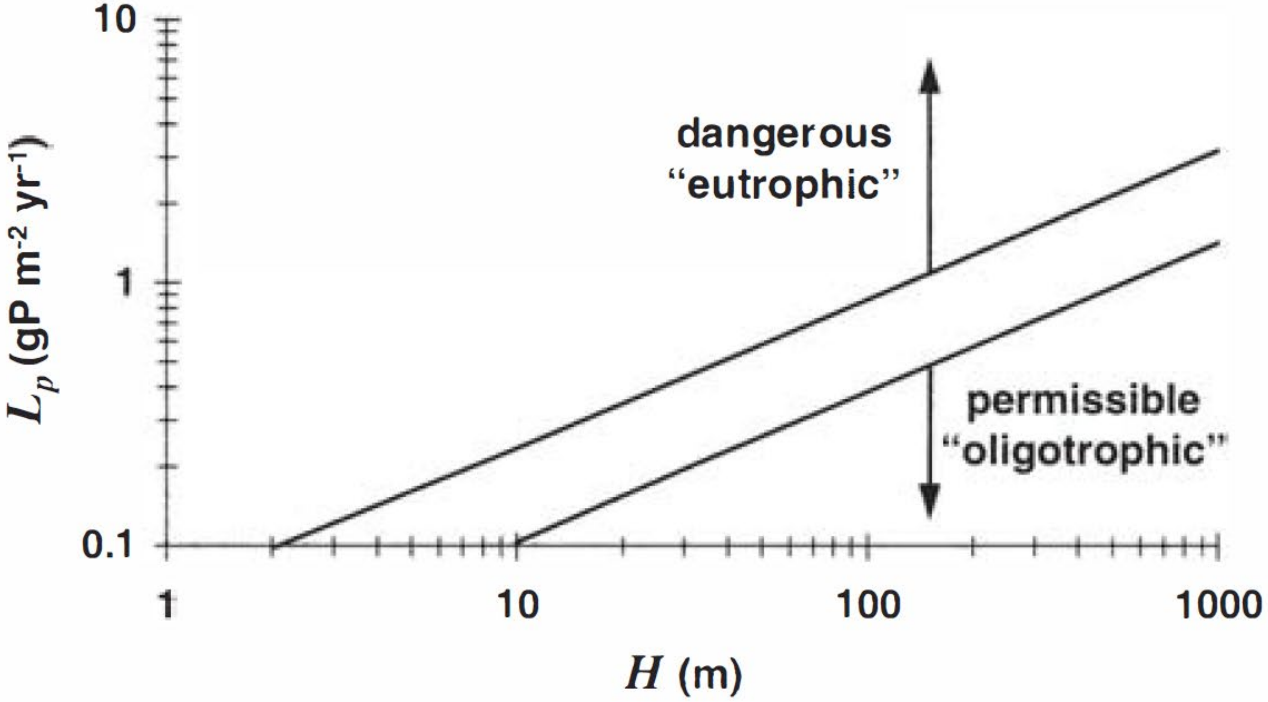



**FIGURE 29.1**
Vollenweider's (1968) loading plot.

# Vollenweider Loading Plots

The phosphorus loading concept is based on the premise that phosphorus is the primary, controllable limiting nutrient of lake and reservoir eutrophication (based on depth observation differences). Richard Vollenweider (1968) developed the first loading plot and one of the early simple empirical models.

Vollenweider compiled areal loadings of total phosphorus $L_p$ ($mgP\ m^{-2}\ yr^{-1}$) and depth H (m) from north temperate lakes from around the world. He labeled them by their trophic status (oligotrophic, mesotrophic, eutrophic)

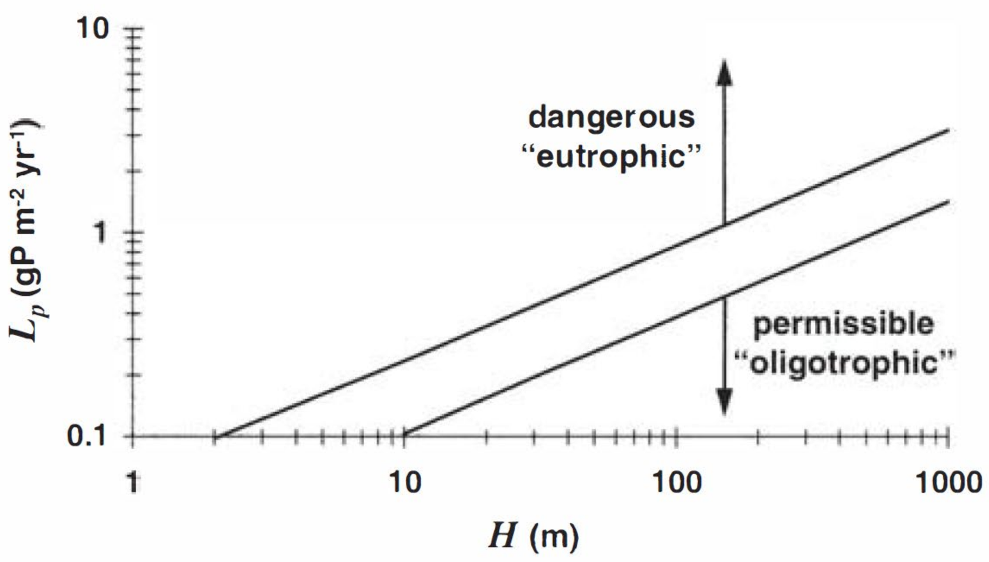



**FIGURE 29.1**
Vollenweider's (1968) loading plot.

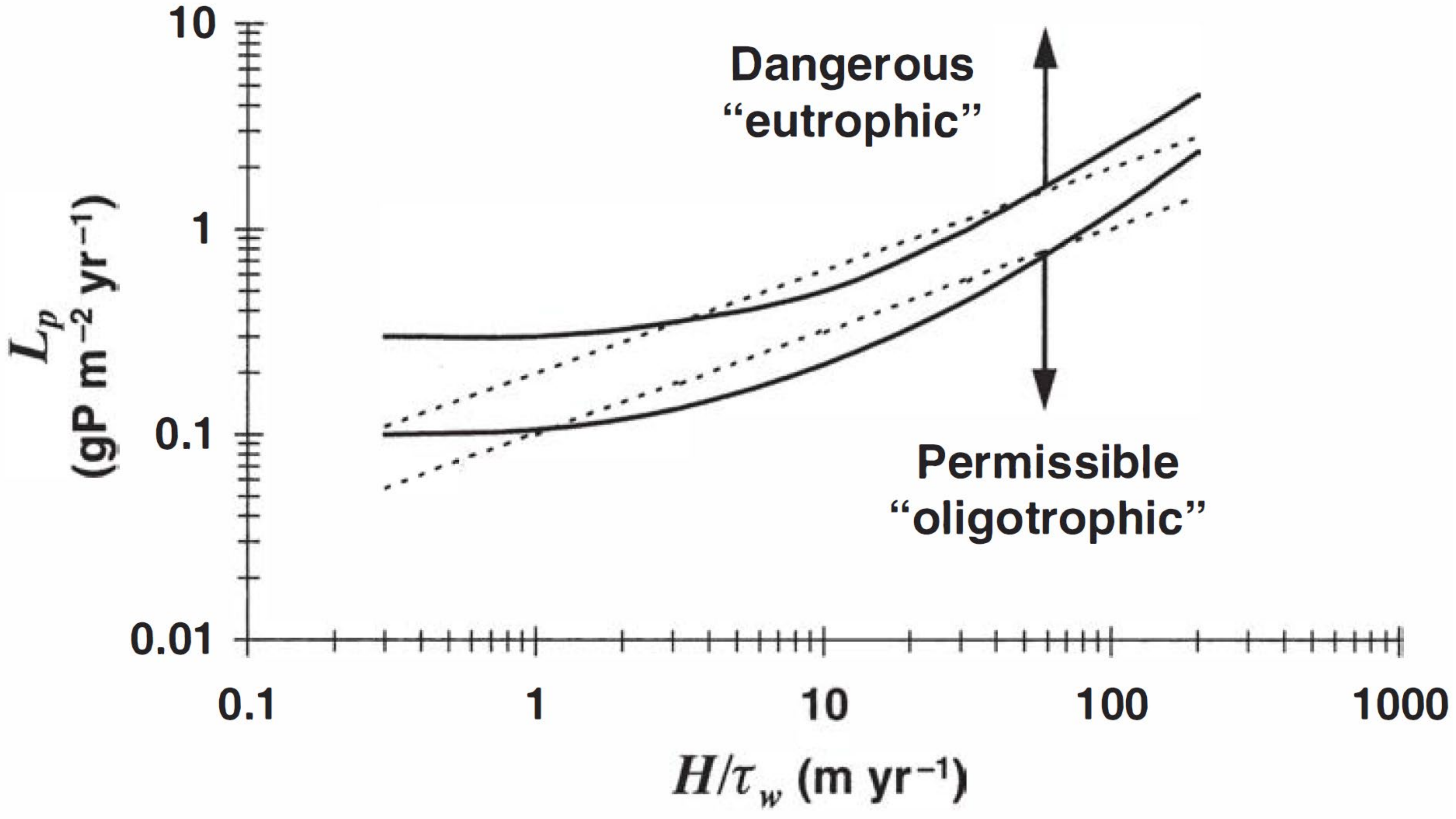



**FIGURE 29.2**
Vollenweider's (1975) loading plot.

# Vollenweider Loading Plots

In 1975 residence time was added as a factor impacting eutrophication. Faster flushing is less susceptible to eutrophication. He suggested a superior fit to the previous plot resulted in curves. These can be used for simulation and wasteload.

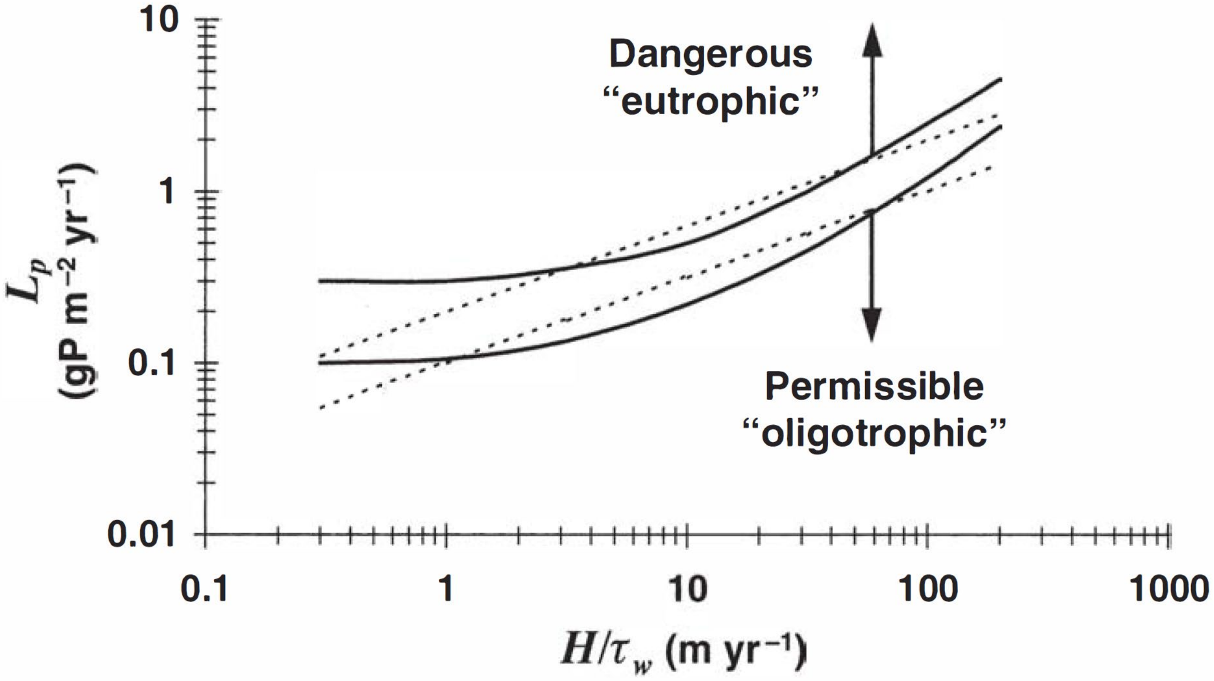



**FIGURE 29.2**
Vollenweider's (1975) loading plot.

The abscissa ($H/\tau_w$) can be shown to be independent of depth.

$$\frac{H}{\tau_w} = \frac{HQ}{V} = \frac{HQ}{HA_s} = \frac{Q}{A_s} = q_s$$

where $q_s$ is the hydraulic overflow rate (m yr$^{-1}$). Historically this has been correlated to sedimentation.

# Budget Models

Early phosphorus loading models used simple mass-balance models, e.g. Vollenweider's application to a well-mixed lake:

$$V\frac{dp}{dt} = W - Qp - k_s V p$$

where *V* = volume ($m^3$)

*p* = total phosphorus concentration (mg $m^{-3}$)

*t* = time (yr)

*W* = total P loading rate (mg $yr^{-1}$)

*Q* = outflow ($m^3$ $yr^{-1}$)

$k_s$ = a first-order settling loss rate ($yr^{-1}$)

# Budget Models

Early phosphorus loading models used simple mass-balance models, e.g. Vollenweider's application to a well-mixed lake:

$$V\frac{dp}{dt} = W - Qp - k_s V p$$

At steady-state, this equation can be solved for:

$$p = \frac{W}{Q + k_s V}$$

Based on the phosphorus budget data (i.e. inputs, outputs, and concentration of phosphorus), the loss rate can be determined as

$$k_s = \frac{W - Qp}{Vp} = \frac{W}{Vp} - \frac{1}{\tau_w}$$

# Budget Models

On the basis of budget calculation, Vollenweider concluded that the loss rate was approximately:

$$k_s = \frac{10}{H}$$

Chapra (1975) suggested that because the loss of phosphorus was due to settling of particulate phosphorus the loss term should be represented by:

$$V\frac{dp}{dt} = W - Qp - vA_s p$$

where $v$ is the settling velocity (m yr$^{-1}$). The steady-state condition can be solved for:

$$p = \frac{W}{Q + vA_s}$$

# Budget Models

The unity between loading plots and budget models can be illustrated by dividing the numerator and denominator of ($p = \frac{W}{Q+vA_s}$) by surface area:

$$p = \frac{L}{q_s + v}$$

$$L = p(q_s + v)$$

Taking the logarithm of this gives:

$$\log L = \log p + \log(q_s + v)$$

# Budget Models

**TABLE 29.1**
**Trophic-state classification based on total phosphorus concentration as well as on other variables reflective of eutrophication**

| Variable | Oligotrophic | Mesotrophic | Eutrophic |
|---|---|---|---|
| Total phosphorus ($\mu$gP $L^{-1}$) | < 10 | 10–20 | > 20 |
| Chlorophyll *a* ($\mu$gChl*a* $L^{-1}$) | < 4 | 4–10 | > 10 |
| Secchi-disk depth (m) | > 4 | 2–4 | < 2 |
| Hypolimnion oxygen (% saturation) | > 80 | 10–80 | < 10 |

# Budget Models

Taking the logarithm of this gives:

$$\log L = \log p + \log(q_s + v)$$

For one extreme (low flushing lakes; small $q_s$) reduces to:

$$\log L = \log p + \log v = constant$$

For high flushing lakes (high $q_s$), the equation reduces to:

$$\log L = \log p + \log q_s$$

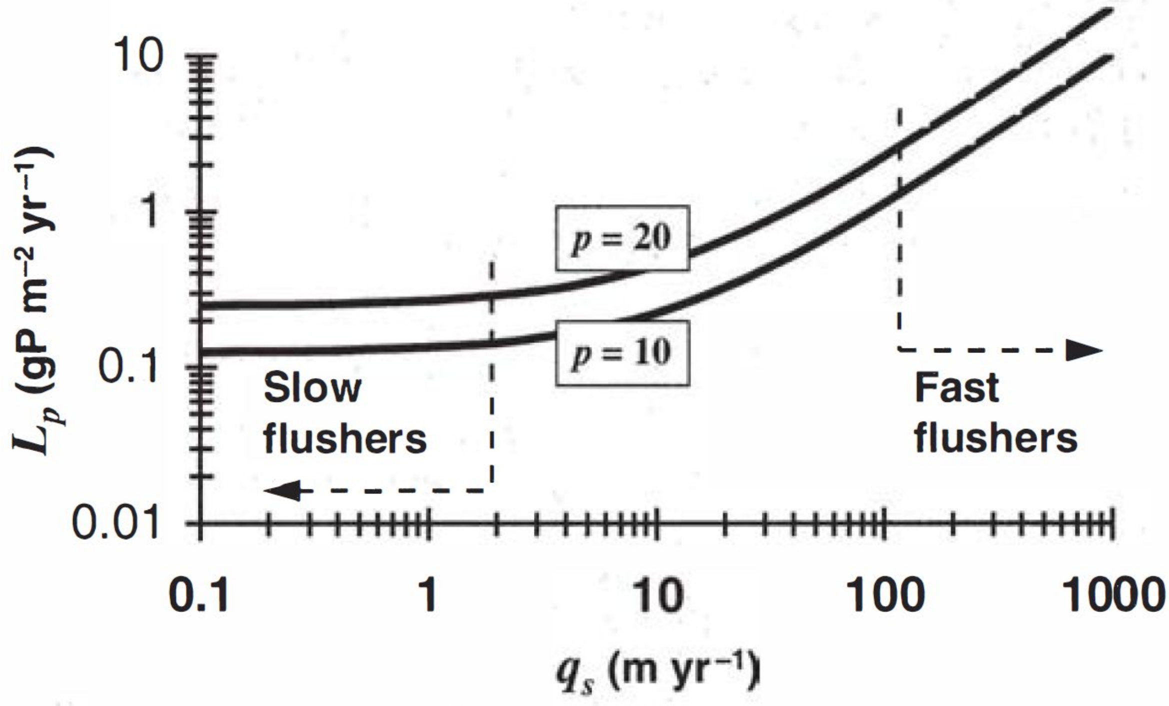



**FIGURE 29.3**
Loading plot derived from a phosphorus budget model (Eq. 29.10 with $v = 12.4$ m yr$^{-1}$).

# Budget Models

Consequently, as assimilation becomes solely dependent on flushing, the curves approach straight lines with a slope of 1.

The 1976 model ($p = \frac{L}{q_s+v}$) can be reformulated and the theoretical developments can provide insights :

$$p = \frac{L}{q_s(1+\sqrt{\tau_w})}$$

Comparing with the 1976 model, we see that settling velocity is:

$$v = q_s\sqrt{\tau_w} = \left(\frac{H}{\tau_w}\right)\sqrt{\tau_w} = \frac{H}{\sqrt{\tau_w}}$$

or a first-order rate of

$$k_s = \frac{1}{\sqrt{\tau_w}}$$

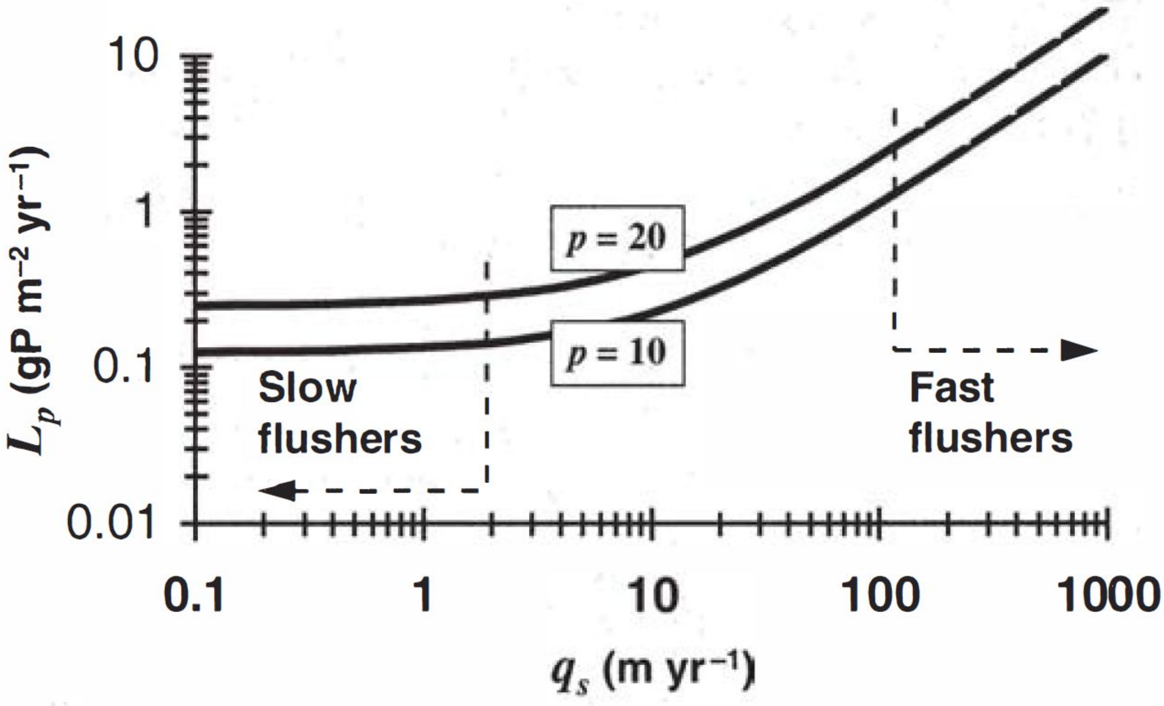



**FIGURE 29.3**
Loading plot derived from a phosphorus budget model (Eq. 29.10 with $v = 12.4$ m yr$^{-1}$).

**What are some differences (benefits/disadvantages) between the loading plot and the budget model?**

**What would you use each type of plot/model for?**

# Trophic-State Correlations

Up to now we have calculated the total phosphorus concentrations and interpreted the resulting levels as indicators of tropic status. Another approach is to use P concentration (or loading) to predict trophic-state variables.

These other variables (Chlorophyll a, Secchi-disk depth, Hypolimnion oxygen demand) provide measures of eutrophication, which are more directly reflective of adverse effects. See empirically-derived correlations.

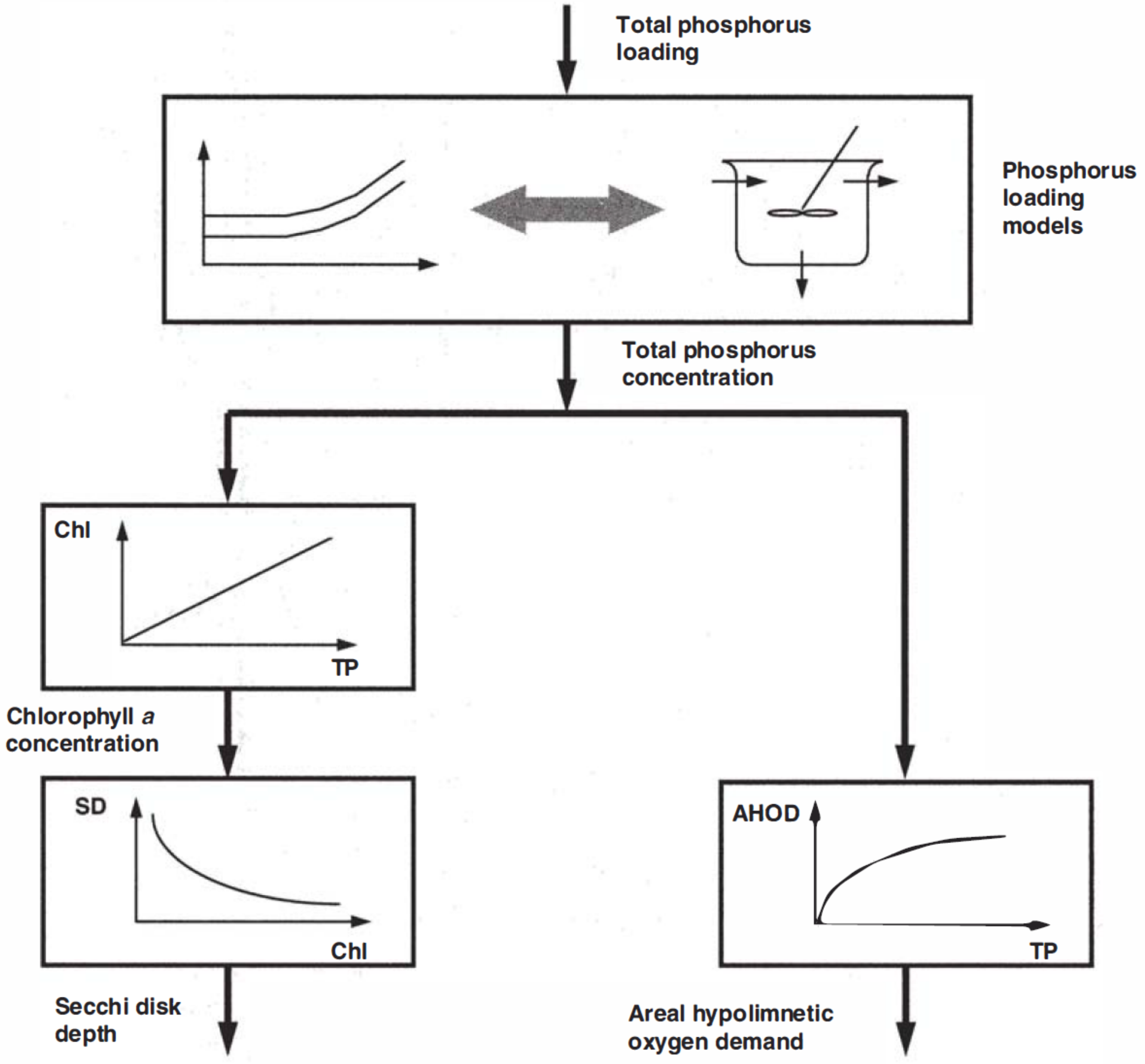



**FIGURE 29.4**
Schematic of approach used by Chapra (1980) to predict trophic state variables based on phosphorus loading model predictions. The approach consists of a number of submodels and correlations that form a hypothesized causal chain that starts with total P concentration predictions based on budget models or loading plots. This concentration is used in conjunction with a series of correlation plots to estimate symptoms of eutrophication such as chlorophyll *a* concentration, Secchi-disk depth, and hypolimnetic oxygen demand.

# Phosphorus-Chlorophyll Correlations

Initial attempts to extend phosphorus loading models correlated Chlorophyll a levels to concentrations of total P. Most based on log-log plot:

Dillon and Rigler (1974):

$$\log(Chla) = 1.449\log(p_v) - 1.136$$

Rast and Lee (1978):

$$\log(Chla) = 0.76\log(p) - 0.259$$

Bartsch and Gakstatter (1978):

$$\log(Chla) = 0.807\log(p) - 0.194$$

where Chl*a* = chlorophyll a concentration (μg $L^{-1}$)

$p$ = total P concentration (μ g $L^{-1}$)

$p_v$ = spring total P concentration (μ g $L^{-1}$)

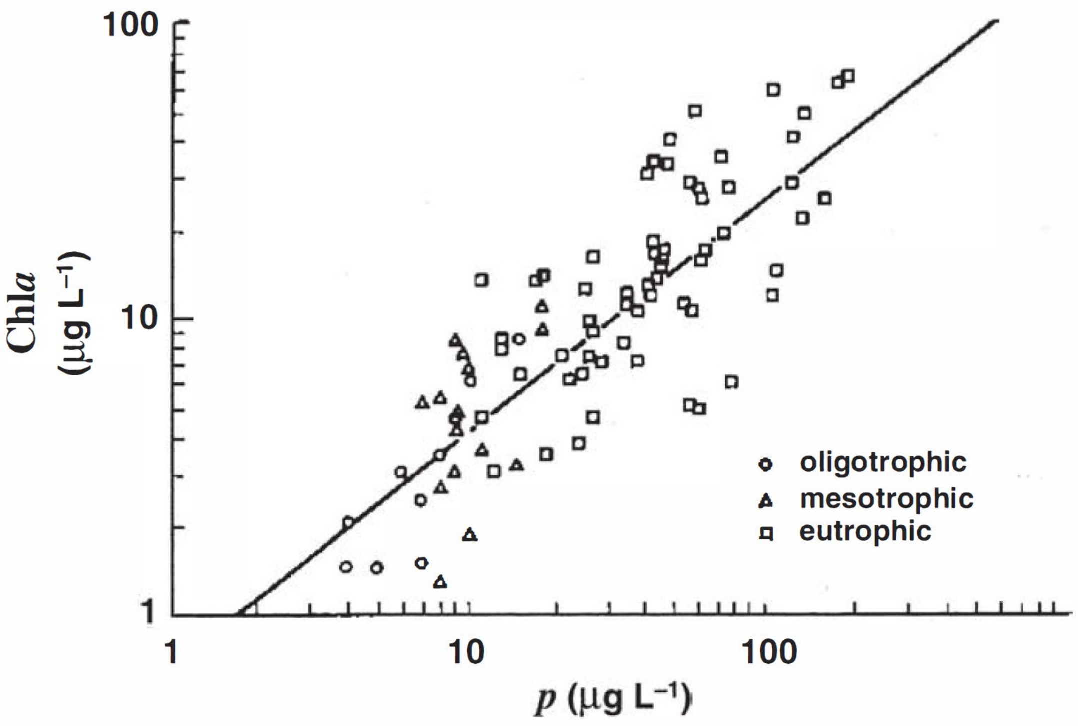



**FIGURE 29.5**
The relationship between chlorophyll *a* and phosphorus in some United States lakes and reservoirs (from Bartsch and Gakstatter 1978).

# Phosphorus-Chlorophyll Correlations

In addition all models are assumed to be appropriate only for phosphorus –limited systems. Smith and Shapiro (1981) have presented a modified correlation that accounts for potential nitrogen limitation,

$$\log(Chla) = 1.55\log(p) - b$$

where

$$b = 1.55\log\left[\frac{6.404}{0.0204(TN{:}TP) + 0.334}\right]$$

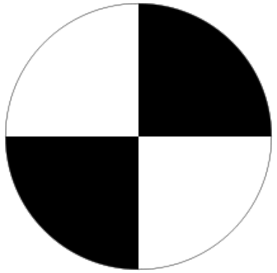
# Chlorophyll-Secchi-Disk Depth Correlations

Secchi-desk depth to chlorophyll levels are also usually started with log-log plots.

$$\log(SD) = -0.473 \log Chl\ a + 0.803$$

where SD = Secchi-disk depth (m). In normal coordinated this equation becomes:

$$SD = 6.35\ Chl\ a^{-0.473}$$



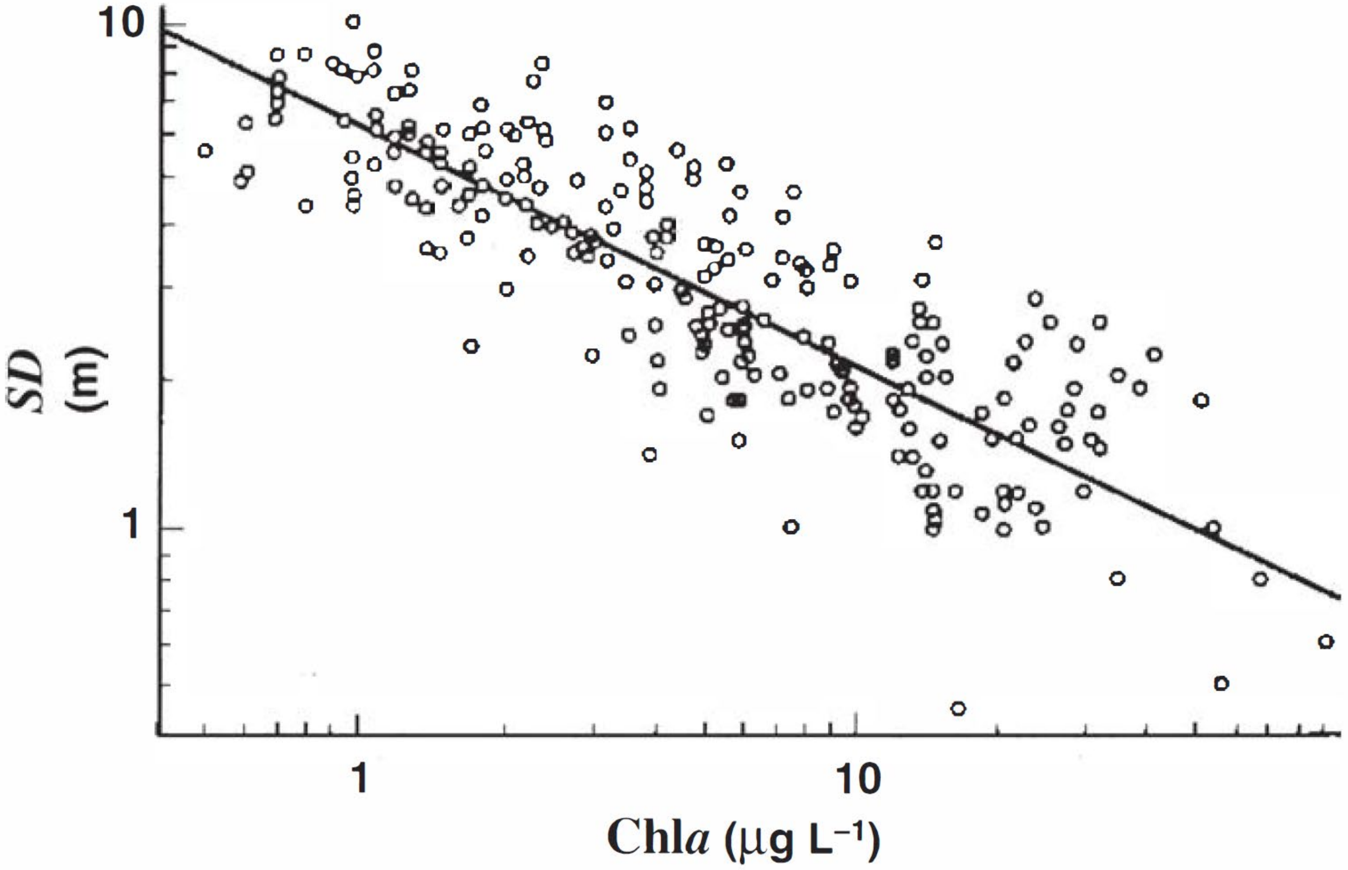



**FIGURE 29.6**
The relationship between Secchi-disk depth and chlorophyll *a* (from Rast and Lee 1978).

# Chlorophyll-Secchi-Disk Depth Correlations

The shape of this relationship can be related to more fundamental measurements by recognizing that light extinction in natural waters is described by:

$$I = I_o e^{-k_e H}$$

where *I* =light at depth H

$I_0$ = light at the surface

$k_e$ = extinction coefficient of water.

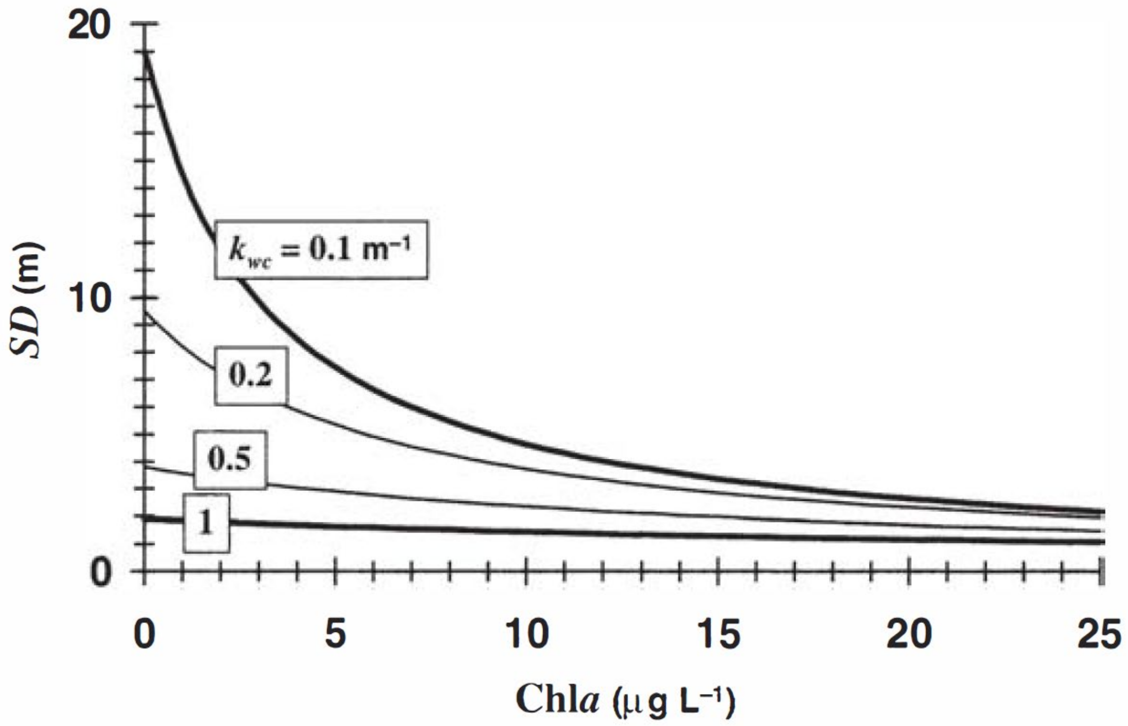



**FIGURE 29.7**
The relationship between Secchi-disk depth and chlorophyll derived from the Beer-Lambert law and light extinction relationships.

# Chlorophyll-Secchi-Disk Depth Correlations

Other investigators have related Secchi-disk depth to light extinction. A rule of thumb is that SD corresponds to the depth at which about 85% of the surface light is extinguished. Thus ($I = I_o e^{-k_e H}$) becomes:

$$0.15 = e^{-k_e SD}$$

The extinction coefficient is:

$$k_e = k_{wc} + \alpha Chla$$

where $k_{wc}$ = extinction due to water, color, non-algal particles ($m^{-1}$) and $\alpha$= coefficient ($\cong 0.035\ L\ \mu g^{-1}\ m^{-1}$).

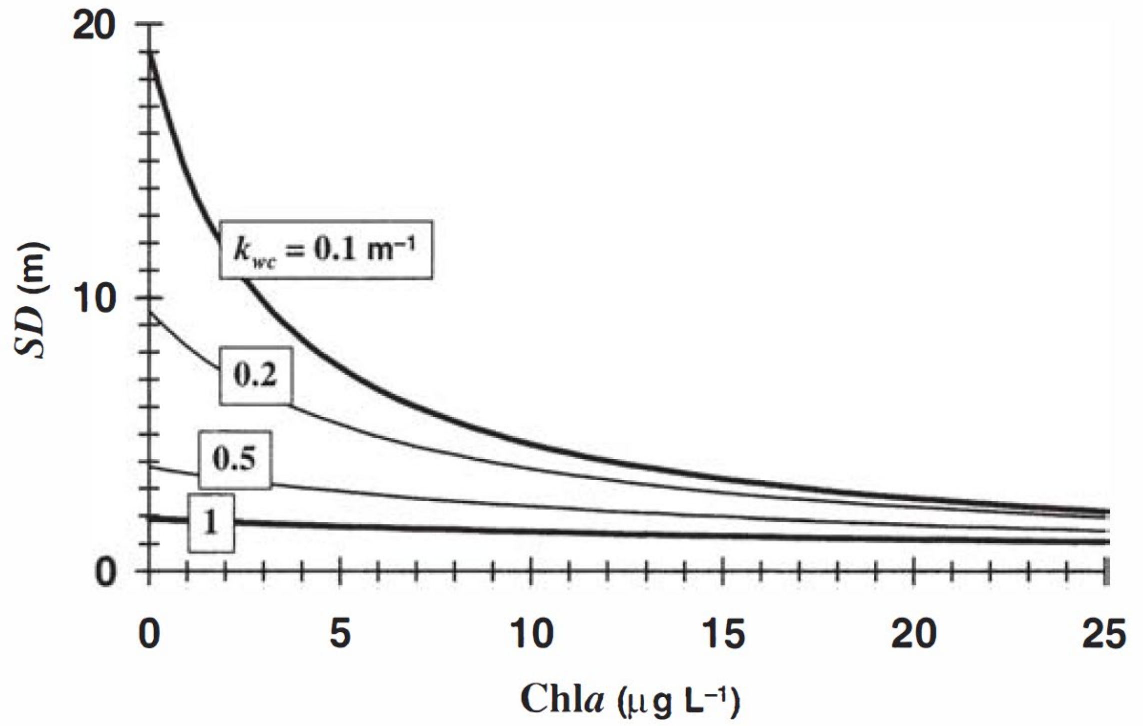



**FIGURE 29.7**
The relationship between Secchi-disk depth and chlorophyll derived from the Beer-Lambert law and light extinction relationships.

# Chlorophyll-Secchi-Disk Depth Correlations

Substituting this relationship and taking the natural logarithm gives :

$$\ln 0.15 = -(k_{wc} + \alpha Chla)SD$$

which can be reformulated as:

$$SD = \frac{1}{1 + \mu Chla} SD_{max}$$

where $\mu = \alpha / k_{wc}$ and $SD_{max} = 1.9/k_{wc}$.

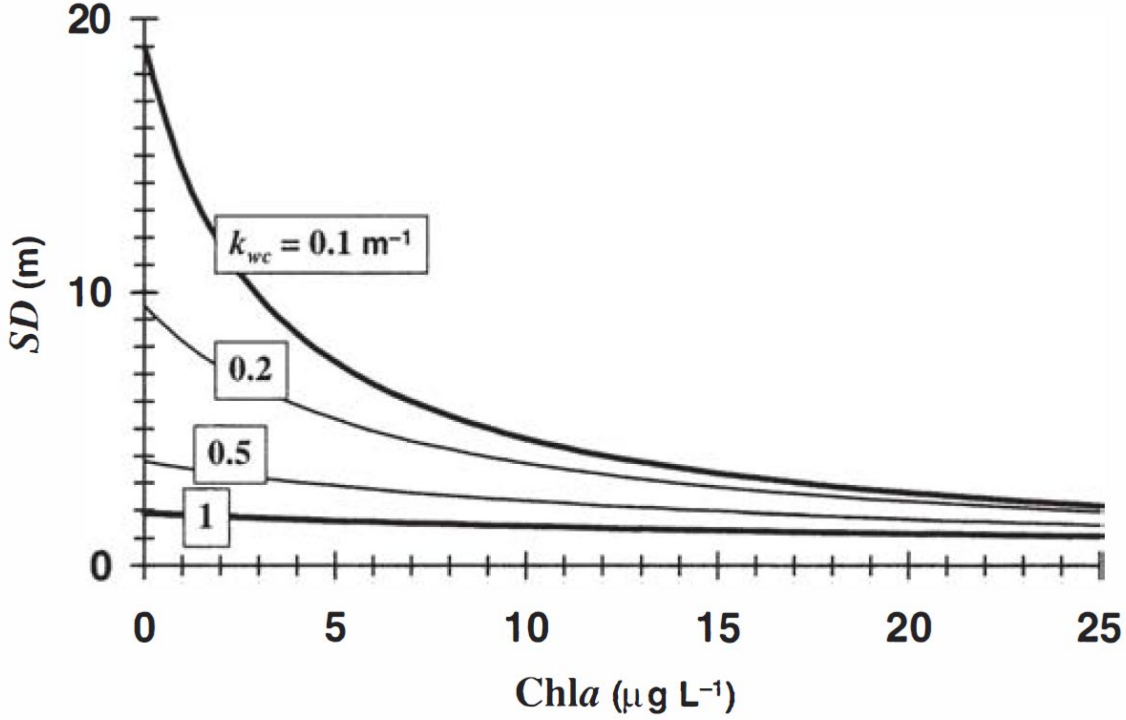



**FIGURE 29.7**
The relationship between Secchi-disk depth and chlorophyll derived from the Beer-Lambert law and light extinction relationships.

# Areal Hypolimnetic Oxygen Demand

Raste and Lee (1978) presented the following correlation to predict the areal hypolimnetic oxygen demand in lakes:

$$\log AHOD = 0.467 \log \left[ \frac{L}{q_s(1+\sqrt{\tau_w})} \right] - 1.07$$

where AHOD = areal hypolimnetic oxygen demand (gO $m^{-2}$ $d^{-1}$), or taking the antilog,

$$AHOD = 0.0851 \left[ \frac{L}{q_s(1+\sqrt{\tau_w})} \right]^{0.467}$$

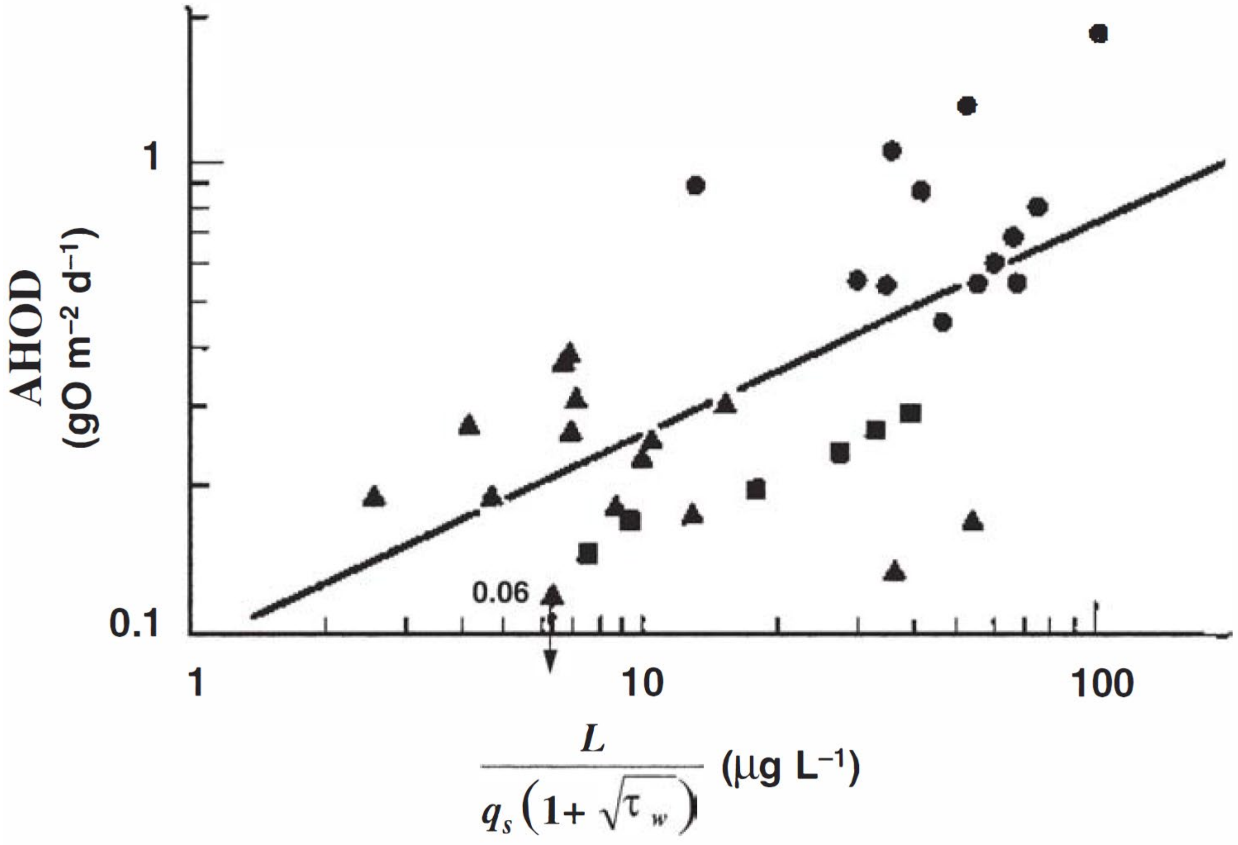



**FIGURE 29.8**
The relationship between areal hypolimnetic oxygen demand and phosphorus loading (Rast and Lee 1978).

# Areal Hypolimnetic Oxygen Demand

This actually correlates AHOD to in-lake total P concentration through ($p = \frac{L}{q_s(1+\sqrt{\tau_w})}$), meaning the direct correlation is :

$$AHOD = 0.086\, p^{0.478}$$

where *p* = mean total P concentration of the lake (µg P $L^{-1}$).

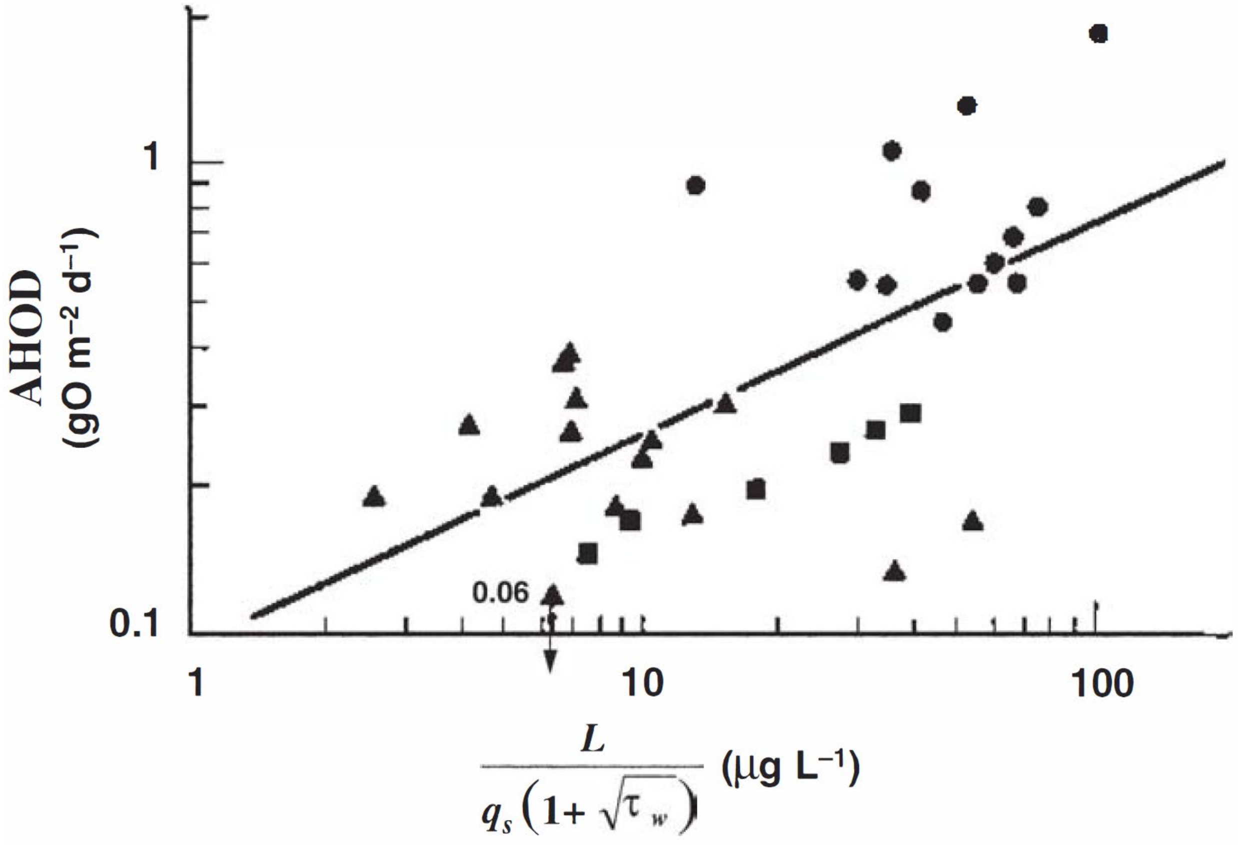



**FIGURE 29.8**
The relationship between areal hypolimnetic oxygen demand and phosphorus loading (Rast and Lee 1978).

# Model/Correlation Summary

The models and correlations have been used because they are easy to apply, yet, they have shortcomings:

- They exhibit large scatter (log-log)
- Prediction error becomes inflated by regional and lake-type variability
- They provide little mechanistic insight into the mechanisms underlying the eutrophication process. Mechanistic models can be extended to assess environmental modifications (e.g. dredging, reaeration, etc) and to guide research and experimentation.