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Elementary Fluid Mechanics CEE 357-02
Fall 2019- December 16
Final Exam

Circle the correct answer or fill in the blank

1. (**2 pts**) The following substance can resist shear: ______________

(a) a liquid
(b) a gas
(c) a solid

2. (**2 pts**) A static column of one or more liquid or gases used to measure pressure differences between two points is called: _____________________.

(a) A barometer
(b) A manometer
(c) A piezometer

3. **(16 pts)** A rigid tank contains 40 pound-mass (1.243 slugs) of air at 20 psi (absolute) and 70 °F. More air is added to the tank until the pressure and temperature rise to 35 psi (absolute) and 90 °F, respectively. Determine the amount of air added to the tank (a) in units of slugs and (b) units of pound-mass.

The gas constant is 1716 $\frac{ft\ lb}{\text{slug°R}}$

$$[(70 + 459.67)\text{°R}] = 529.67\ R$$

$$\rho = \frac{p}{RT} \qquad V = \frac{m_1 R T_1}{p_1}$$

$$V = \frac{(1.243\ slugs)\left(1716\,\frac{ft\ lb}{\text{slug°R}}\right)[(70 + 459.67)\text{°R}]}{\left(20\,\frac{lb}{in^2}\right) * (144\,\frac{in^2}{ft^2})}$$

$$V = 392.36\ ft^3 \qquad (4pts)$$

$$m_2 = \frac{p_2 V}{R T_2}$$

$$m_2 = \frac{\left(35\,\frac{lb}{in^2}\right) * (144\,\frac{in^2}{ft^2})(392.36\ ft^3)}{\left(1716\,\frac{ft\ lb}{\text{slug°R}}\right)[(90 + 459.67)\text{°R}]} \qquad (4pts)$$

$$m_2 = 2.097\ slugs$$

(a) $\quad air\ added = m_2 - m_1 = 2.097 - 1.243 = 0.853\ slugs \qquad (4pts)$

(b) $\quad air\ added\ in\ lbm = 0.853\ slugs \times \frac{32.174\ lbm}{1\ slug} = 27.453 \qquad (4pts)$

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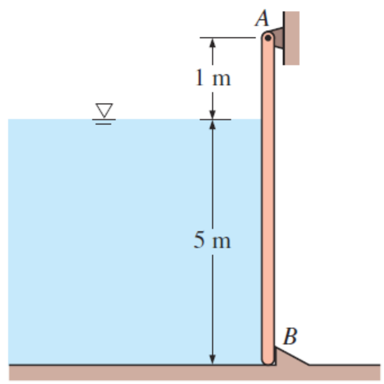
4. (**15pts**) A 6-m-high, 5-m-wide rectangular plate blocks the end of a 5-m-deep freshwater channel, as shown in the Figure below. The plate is hinged about a horizontal axis along its upper edge through a point A and is restrained from opening by a fixed ridge at point B. Determine (a) the average pressure on the surface of the wall at the centroid of the surface, (b) the resultant hydrostatic force acting on the left side of the wall for the entire area of the wall, and (c) where this line of action passes through (i.e. the center of pressure) as measured in terms of the depth below the free surface on the left side of the wall (in meters).



$$(a) \quad p_{ave} = p_{CG} = \gamma\, h_{CG} = (9790\ N/m^3)(5\ m/2) = 24,475\ N/m^2 \quad (5pts)$$

$$(b) \quad F_R = p_{CG}\, A = (24,475\ N/m^2)(6m \times 5\ m) = 734{,}250\ N = 734.25\ KN \quad (5\ pts)$$

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A}$$

$$I_{xx} = \frac{bL^3}{12} = \frac{(5)(5^3)}{12} = \frac{625}{12} = 52.0833$$

$$y_{CP} = -\frac{(52.083)\sin 90°}{(2.5\ m)(5\ m\ \times 5m)} = -0.8333$$

$$Depth = h_{CG} + y_{CP} = (-2.5\ m) + (-0.833\ m) = -3.33\ m \quad or$$

$$3.33\ m\ below\ the\ surface \qquad (5\ pts)$$

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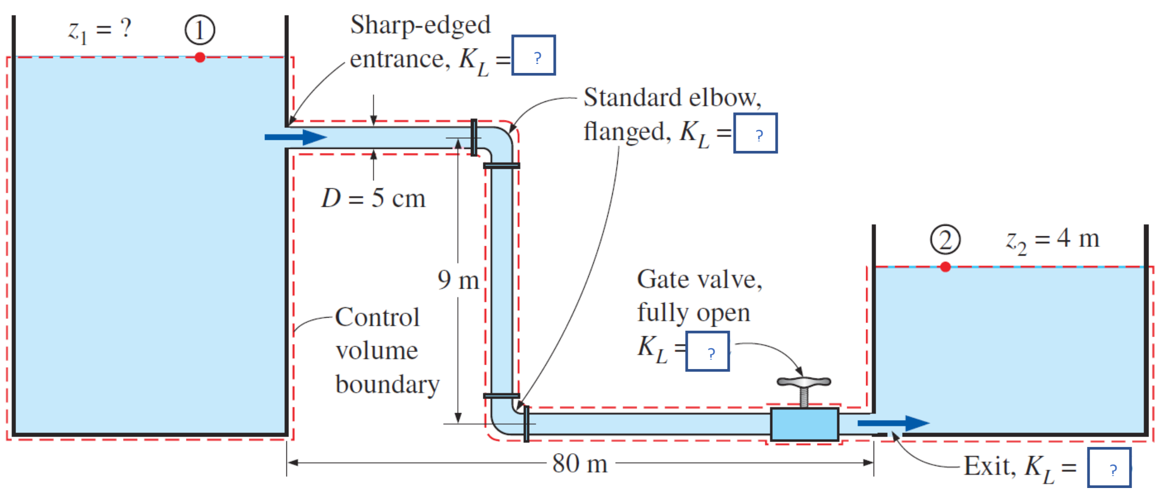
5. (**15pts**) Water at 10°C flows from a large reservoir to a smaller one through a 5-cm-diameter cast iron piping system, as shown below. Determine the elevation Z_1 for a flow rate of 6 Liters/s.



$$\frac{p_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_L \qquad \rightarrow \quad z_1 = z_2 + h_L$$

where

$$h_L = h_f+\sum h_m = \left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g}$$

$$V = \frac{Q}{A_c} = \frac{0.006\ m^3/s}{\pi D^2/4} = \frac{0.006\ m^3/s}{\pi(0.05)^2/4} = 3.06\ m/s$$

$$Re_d = \frac{\rho V D}{\mu} = \frac{(999.7\ kg/m^3)(3.06\ m/s)(0.05\ m)}{1.307\times10^{-3}\ kg/m\cdot s} = 117{,}000 \qquad (5pts)$$

The flow is turbulent since Re> 4000. Noting that e/D = 0.00026/0.05 = 0.0052, the friction factor is determined from the Colebrook equation (or the Moody chart),

$$\frac{1}{f^{1/2}} = -2.0\log_{10}\left(\frac{\varepsilon/d}{3.7}+\frac{2.51}{Re_d f^{1/2}}\right) \qquad or \quad f=\left[1/\left(-2\log_{10}\left(\frac{\varepsilon/d}{3.7}+\frac{2.51}{Re_d f^{1/2}}\right)\right)\right]^2$$

$$f = 0.0315 \qquad (5pts)$$

$$\sum K_L = K_{entrance}+2K_{elbow}+K_{valve}+K_{exit}$$
$$= 0.5+2\times0.3+0.35+1.0 = 2.45$$

$$h_L = \left(f\frac{L}{D}+\sum K_L\right)\frac{V^2}{2g} = \left(0.0315\frac{89m}{0.05m}+2.45\right)\frac{(3.06\ m/s)^2}{2\ (9.81\ m/s^2)} = 27.93\ m$$

$$z_1 = z_2 + h_L$$
$$z_1 = 4 + 27.93 = 31.93\ m \qquad (5pts)$$

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Circle the correct answer or fill in the blank

6. (**2pt**) ___________, _________________, and_____________, are the three reversible forms of energy in the Bernoulli principle.

Pressure work, kinetic energy, potential energy
(pressure head, velocity head, elevation head)

7. (**2 pt**) An elementary wave traveling on the surface of a fluid within a subcritical regime would ____________relative to the flow of the fluid.

(a) flow downstream
(b) remain stationary
(c) flow upstream

8. (**2 pt**) The best of all possible channel cross-sections (maximizing R_h, minimizing P) is _______.

(a) a square with the relation b = 2y
(b) a trapezoid
(c) a semi-circle

9. (**2 pt**) Supercritical flow requires a height (y) above the channel bottom that is _______.

(a) greater than the critical depth (yc)
(a) less than the critical depth (yc)
(a) equal to the critical depth (yc)

10. (**2 pt**) A supercritical flow regime will__________ when flowing over a depression:

(a) stay the same depth
(b) decrease in depth
(c) increase in depth

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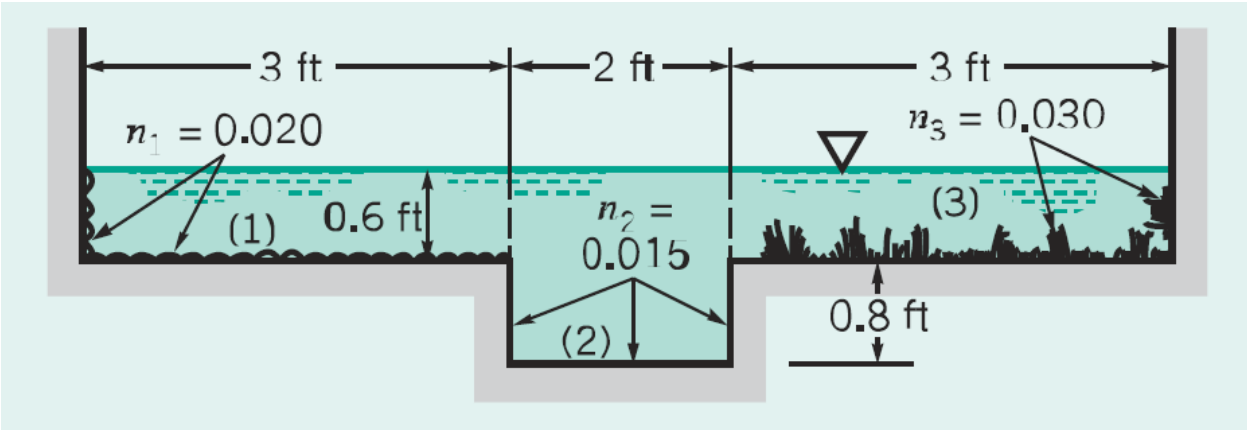
Solve and show your work.

11. Uniform flow in a Channel. (**20 pts**)

Water flows along the drainage canal having properties shown in the Figure. The bottom slope is S_o = 1ft/250 ft = 0.004, with variable Manning roughness coefficients (n_1= 0.020, n_2 = 0.015, n_3= 0.030) for the three sub-sections of flow in the channel.

Estimate the flow rate in ft^3/s.



Divide the cross section into three subsections as indicated in the Figure and write:

$Q = Q_1 + Q_2 + Q_3$, where for each section :

$$Q_i = \frac{1.49}{n_i} A_i R_{hi}^{2/3} S_o^{1/2}$$

The appropriate values of $A_i, P_i, R_{hi}, and\ n_i$ are:

$$A_1 = (3\,ft)(0.6\,ft) = 1.8\,ft^2\,; \quad A_2 = (2\,ft)(0.8 + 0.6) = 2.8\,ft^2\,; \quad A_3 = (3\,ft)(0.6\,ft) = 1.8\,ft^2$$

(3pts)

$$P_1 = (3\,ft) + (0.6\,ft) = 3.6\,ft;\ P_2 = (2\,ft) + 2(0.8) = 3.6\,ft;\ P_3 = (3\,ft) + (0.6\,ft) = 3.6\,ft$$

(3pts)

Note that the wetted perimeter P_2 is only the channel portion …

$$R_{h1} = \frac{1.8\,ft^2}{3.6\,ft} = 0.50\,ft;\ \ R_{h2} = \frac{2.8\,ft^2}{3.6\,ft} = 0.778\,ft;\ R_{h3} = \frac{1.8\,ft^2}{3.6\,ft} = 0.50\,ft$$

(3pts)

$$n_1 = 0.02\,;\ n_2 = 0.015;\ \ n_3 = 0.030$$

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$$Q = Q_1 + Q_2 + Q_3$$

$$Q = 1.49\ (0.004)^{1/2}\ \times \left[\frac{(1.8ft^2)(0.5ft)^{2/3}}{0.020} + \frac{(2.8ft^2)(0.778ft)^{2/3}}{0.015} + \frac{(1.8ft^2)(0.5ft)^{2/3}}{0.030}\right]$$

$$(5\ pts)$$

$$Q_{total} =\ 1.49\ (0.0632) \times [\ 56.70 +\ 157.87 +\ 37.80]$$

$$Q_{total} =\ 1.49\ (0.0447) \times [\ 252.37] = 23.78\ \ ft^3/s \qquad\qquad (6\ pts)$$

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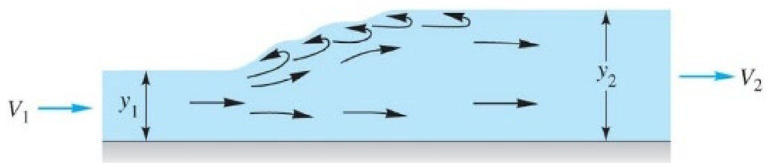
Solve and show your work.

12. Hydraulic Jump **(20 Pts)**

Water flowing in a wide channel 25 cm deep suddenly jumps to a depth of 1.5 m.

(a) Estimate the downstream Froude number (Fr_2), Round the final answer to three decimal places.
(b) Estimate the flow rate per unit width (q_1), Round the final answer to two decimal places.
(c) Estimate the critical depth (y_c), Round the final answer to two decimal places.
(d) Estimate the percentage of dissipation (%). Round the final answer to two decimal places.



$$\frac{2y_2}{y_1} = -1 + (1+8Fr_1^2)^{1/2}$$

$$\frac{2(1.5\,m)}{(0.25m)} = -1 + (1+8Fr_1^2)^{1/2}$$

$$Fr_1^2 = \left[\left(\frac{2(1.5\,m)}{(0.25m)}+1\right)^2 - 1\right]/8$$

$$Fr_1 = 4.58$$

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} \quad \rightarrow \quad V_1 = Fr_1\sqrt{gy_1} = 4.58\sqrt{9.81\,(0.25m)} = 7.177$$

$$V_1 = \frac{V_1 y_1}{y_2} = \frac{(7.177)(0.25m)}{1.5m} = 1.196\;m/s$$

$$(a) \qquad Fr_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{1.196\frac{m}{s}}{\sqrt{9.81\,(1.5m)}} = 0.312 \qquad (4pts)$$

$$(b) \qquad q = V_1 y_1 = \left(1.196\frac{m}{s}\right)(0.25\,m) = 1.79\;m^2/s \qquad (4pts)$$

$$(c) \qquad y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(1.79)^2}{9.81}\right)^{1/3} = 0.69\,m \qquad (4pts)$$

$$(d) \qquad E_1 = y_1 + \frac{V_1^2}{2g} = \;2.875\,m; \qquad h_f = \frac{(y_2-y_1)^3}{4y_1y_2} = \frac{(1.5-0.25)^3}{4*(0.25)(1.5)}\;1.30\,m$$

$$\%\,dissipation = \frac{h_f}{E_1} = \frac{1.3}{2.875} = 45.29\,\% \qquad (4pts)$$

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---------------BONUS----------------------

Bonus (2 points): A pump from the family of Fig 11.8 (Non-dimensional plot of Performance) has D = 28 in and n = 18,000 r/min. Estimate the discharge (Q*) for water at 60F (density =1.94 slugs/ft^3) at its best efficiency.

n = 18,000 r/min = 300 r/s

$$Q^* = C_Q n D^3 = (0.115)(300\ r/s)\left(\frac{28in}{\frac{12in}{ft}}\right)^3 = 483.3\frac{ft^3}{s} = 196,699\ gal/min$$

Bonus (2 points): A cast iron duct (n =0.013) of diameter 3.1 m is flowing half-full at 36 m^3/s. Determine the slope of this duct if the flow is uniform. Round answer to four decimal places.

$$Q = \frac{1.0}{n} A R_h^{2/3} S_o^{1/2}$$

$$P = \pi R = \pi(1.55m)$$

$$R_h = \frac{A}{P} = \frac{\pi R^2/2}{\pi R} = 0.775$$

$$S_o^{1/2} = \frac{Qn}{AR_h^{2/3}} = \frac{(36\ m^3/s)(0.013)}{(\frac{\pi(1.55)^2}{2})(0.775^{2/3})} = 0.1470$$

$$S_o = 0.16096^2 = 0.0216$$

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