March 30, 2020

1. Reflection: Which of the previous topics have helped to understand modeling of P loading? Which additional topics could complement this topic for next year's class?

2. Sediment-Water Interactions
3. Application Shagawa Lakes
4. Simplest Seasonal Approach
5. Model Formulation
6. Application to Lake Ontario

Sediment-Water Interactions

Bottom sediments have long been acknowledged as a potential source of phosphorus to the overlying waters of lakes and impoundments.

Sediment feedback could have significant impact on recovery of these systems, especially in shallow lakes (or those with anaerobic hypolimnia).

Here we demonstrate semiempirical formulations used to simulate sediment feedbacks in conjunction with simple total P budgets for water and sediments.

Sediment-Water Model

We develop a simple modeling framework to address this problem for stratified lakes.

There are two components:

-a total phosphorus budget

-a model of hypolimnetic oxygen deficit.

Total Phosphorus Model

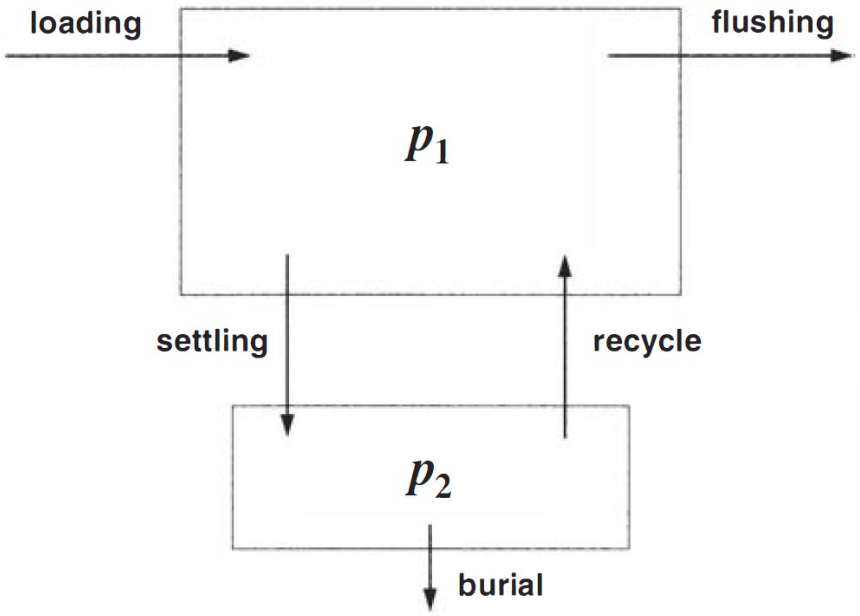


FIGURE 29.9
Schematic diagram of a phosphorus budget model for a lake underlain by sediments.

$$V_1 \frac{dp_1}{dt} = W - Qp_1 - v_s A_s p_1 + v_r A_s p_2$$

$$V_2 \frac{dp_2}{dt} = v_s A_s p_1 - v_r A_s p_2 - v_b A_s p_2$$

where subscript 1 and 2 designate water and enriched surface sediment layer, respectively.

v_s = settling velocity of P (m yr^{-1})

A_s = surface area of deposition zone (m^2)

v_r = recycle mass-transfer coefficient from the sediments to the water (m yr^{-1})

v_b = burial mass-transfer coefficient (m yr^{-1})

Hypolimnetic oxygen model

A zero-order model is employed here to simulate hypolimnetic oxygen during periods when the lake is stratified,

$$o_h = o_i - \frac{AHOD}{H_h}(t - t_s)$$

where o_h = hypolimnetic dissolved oxygen level (g m^{-3})

o_i = initial oxygen concentration at onset of stratification (g m^{-3})

AHOD = areal hypolimnetic oxygen demand (g m^{-2} d^{-1})

H_h = average hypolimnion thickness (m)

t = time (d)

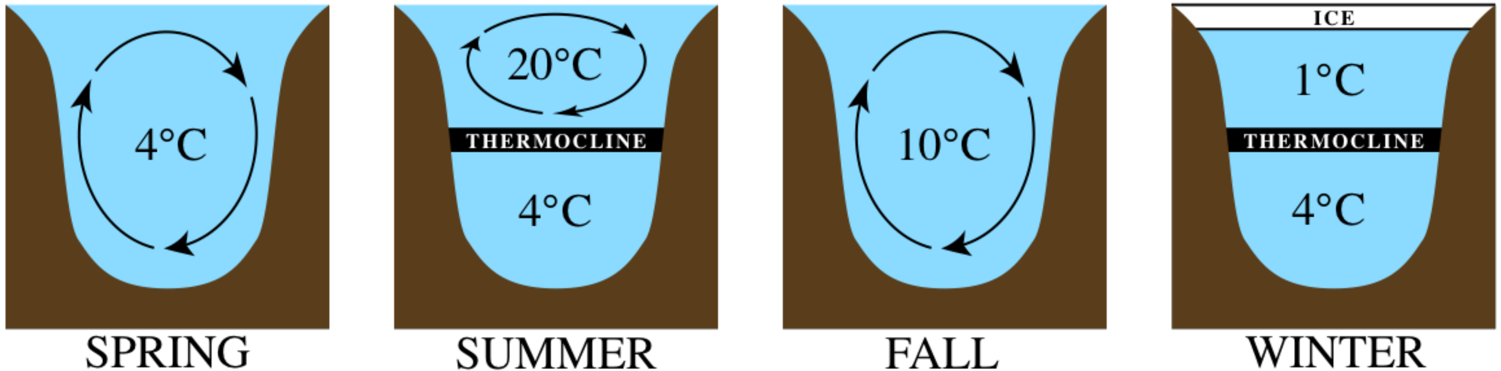
t_s = time of onset of stratification

Hypolimnetic oxygen model

To predict AHOD simple means are available to estimate this quantity. E.g. ($AHOD = 0.086\, p^{0.478}$). For dimictic lakes, an AHOD can also be exerted during winter inverse stratification.

$$AHOD_w = AHOD_s\, 1.08^{T_w - T_s}$$

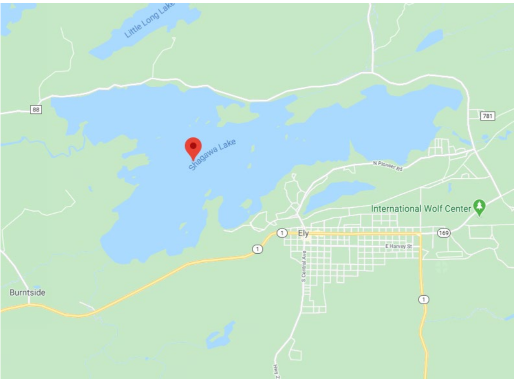
where T_s = temperature (°C) at which summer $AHOD_s$ is measured and T_w is the temperature (°C) corresponding to the winter $AHOD_w$.



Application: Shagawa Lake

Shagawa Lake, Minnesota is a location that can be used to illustrate the model. Here, sediment feedback was recognized as being important. In the early 1970s, significant nutrient load reductions were observed after many years of pollution.

Model calibration is based on the extensive data reported by Larsen, Malueg et al. 1976-1981.



Parameter values were determined by calibrating for the pretreatment period for which data is available (1967-1972). The following example assumes the lake is at steady-state.

Application: Shagawa Lake

EXAMPLE 29.1. CALIBRATION OF SEDIMENT-WATER TOTAL P MODEL. Data for Shagawa Lake from 1967 through 1972 are summarized in Tables 29.2 and 29.3. Use this data to calibrate the sediment-water total P model. Specifically, estimate (*a*) the burial and (*b*) the recycle velocities.

TABLE 29.2
Data for Shagawa Lake (1967 to 1972)

Parameter	Symbol	Value	Units
Volume	V_1	53×10^6	m^3
Surface area	A_1	9.6×10^6	m^2
Mean depth	H_1	5.5	m
Hypolimnion thickness	H_h	2.2	m
Deposition zone area	A_2	4.8×10^6	m^2
Surface sediment thickness	H_2	10	cm
Total P loading	W_{in}	6692×10^6	mg yr^{-1}
Total P outflow	W_{out}	4763×10^6	mg yr^{-1}
Mean water P concentration	p_1	56.3	mg m^{-3}
Mean sediment P concentration	p_2	500,000	mg m^{-3}
Hypolimnion temperature—summer	$T_{h,s}$	15	°C
Hypolimnion temperature—winter	$T_{h,w}$	4	°C
Total P settling velocity	v_s	42.2	m yr^{-1}
Summer initial hypolimnetic DO	$DO_{i,s}$	8	mg L^{-1}
Winter initial hypolimnetic DO	$DO_{i,w}$	8	mg L^{-1}

TABLE 29.3
Stratification data for Shagawa Lake (1967 to 1972)

Event	Day
Start of spring mixed period	120
Start of summer stratification	150
Start of fall mixed period	255
Start of winter stratification	320

Application: Shagawa Lake

$$V_1 \frac{dp_1}{dt} = W - Qp_1 - v_s A_2 p_1 + v_r A_2 p_2$$

$$V_2 \frac{dp_2}{dt} = v_s A_2 p_1 - v_r A_2 p_2 - v_b A_2 p_2$$

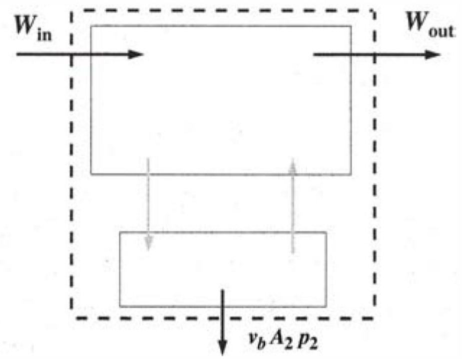


FIGURE E29.1-1

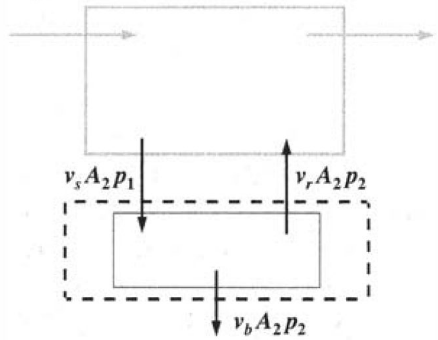


FIGURE E29.1-2

TABLE 29.2
Data for Shagawa Lake (1967 to 1972)

Parameter	Symbol	Value	Units
Volume	V_1	53×10^6	m^3
Surface area	A_1	9.6×10^6	m^2
Mean depth	H_1	5.5	m
Hypolimnion thickness	H_h	2.2	m
Deposition zone area	A_2	4.8×10^6	m^2
Surface sediment thickness	H_2	10	cm
Total P loading	W_{in}	6692×10^6	$mg\ yr^{-1}$
Total P outflow	W_{out}	4763×10^6	$mg\ yr^{-1}$
Mean water P concentration	p_1	56.3	$mg\ m^{-3}$
Mean sediment P concentration	p_2	500,000	$mg\ m^{-3}$
Hypolimnion temperature—summer	$T_{h,s}$	15	°C
Hypolimnion temperature—winter	$T_{h,w}$	4	°C
Total P settling velocity	v_s	42.2	$m\ yr^{-1}$
Summer initial hypolimnetic DO	$DO_{i,s}$	8	$mg\ L^{-1}$
Winter initial hypolimnetic DO	$DO_{i,w}$	8	$mg\ L^{-1}$

Application: Shagawa Lake

This figure shows the simulation results for a single year (steady-state) calibration period. Phosphorus increases due to heightened release rate when oxygen falls below 1.5 mg L^{-1}.

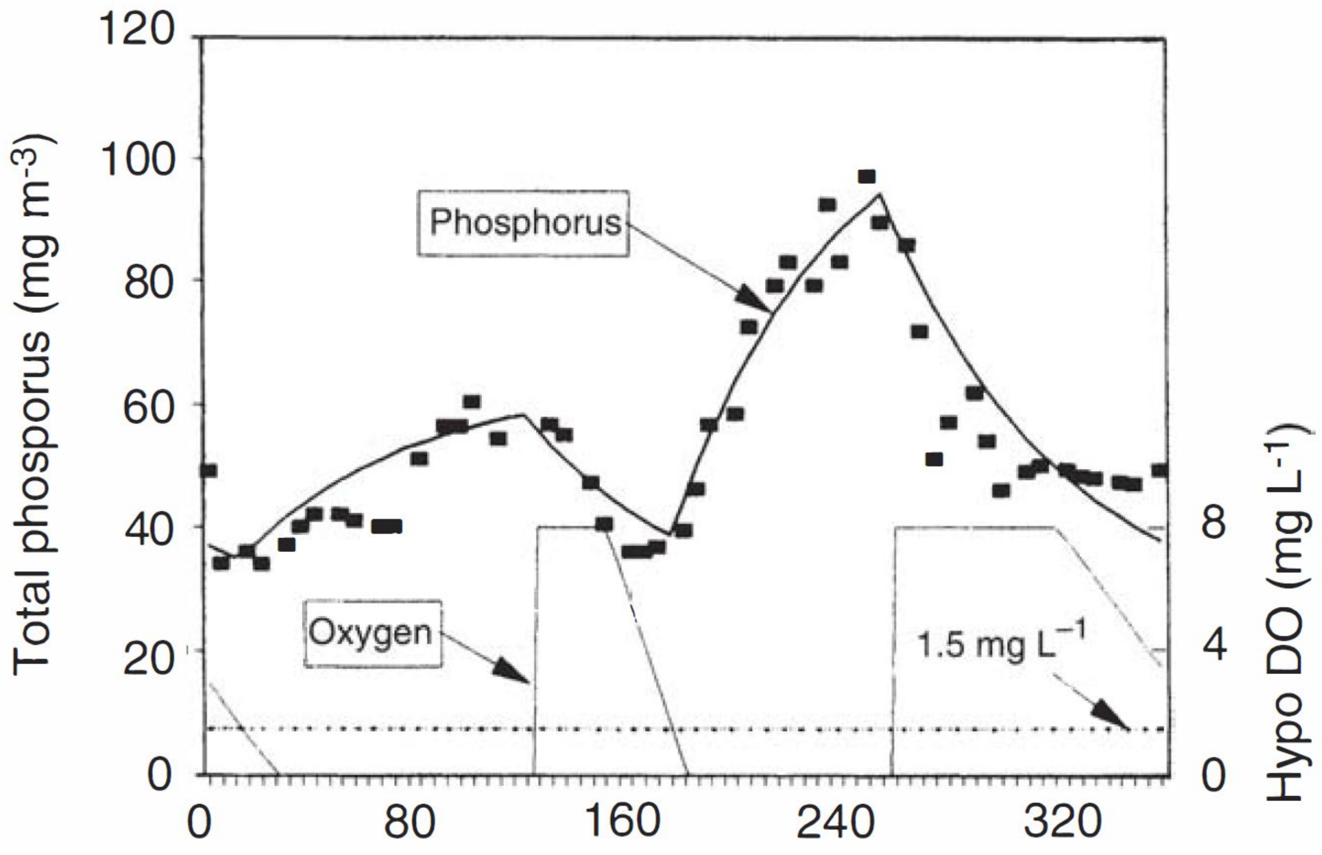


FIGURE 29.10
Plot of phosphorus and oxygen in the pretreatment period (1967–1972) for Shagawa Lake. The data are for 1972 (Larsen et al. 1979).

Application: Shagawa Lake

A long-term simulation was performed from 1880 to 2000. Average flows were used for all years except 1967 to 1979. An idealized long-term loading scenario was developed. Before 1890 and after 1979, an average “natural” loading of 1311 kg yr^{-1} was assumed to apply.

In 1890, the town was established, stepwise loading until 1960.

1973 load decreased, then slowed.

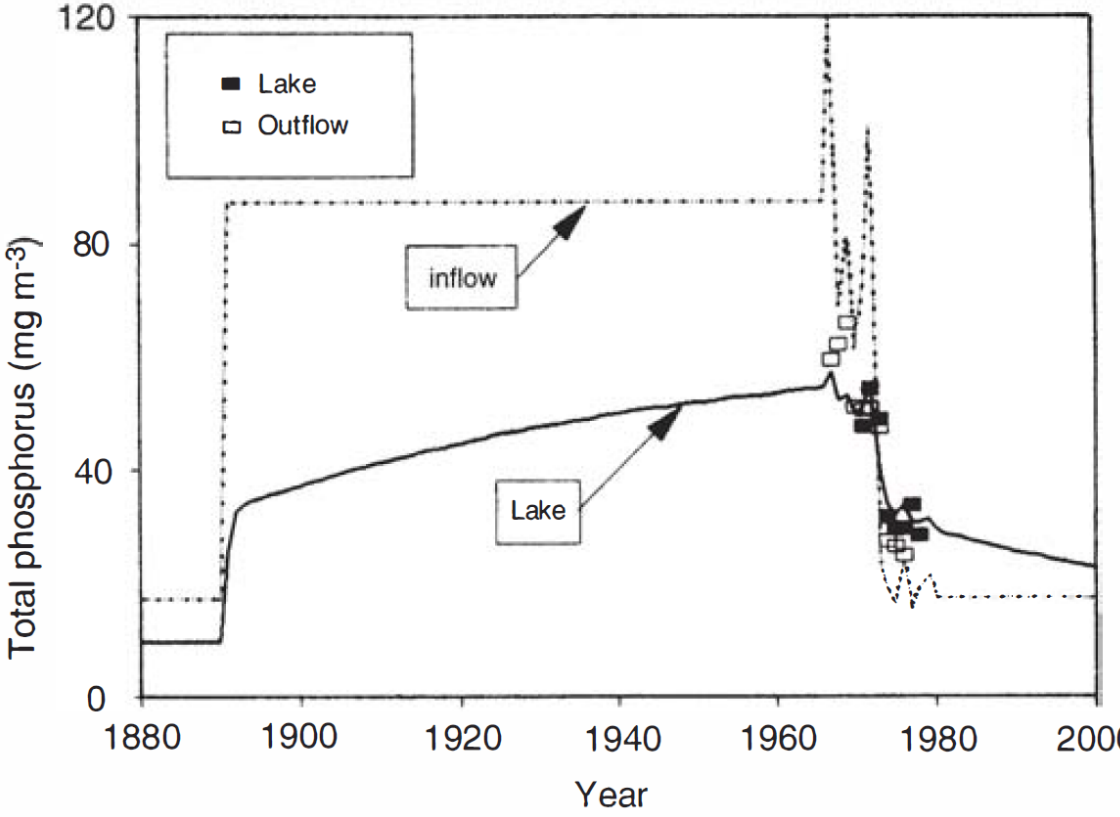


FIGURE 29.11
Long-term total phosphorus concentration for Shagawa Lake as simulated by the phosphorus-oxygen model (thick line). A plot of inflow concentration is superimposed (thin line) for comparison.

Application: Shagawa Lake

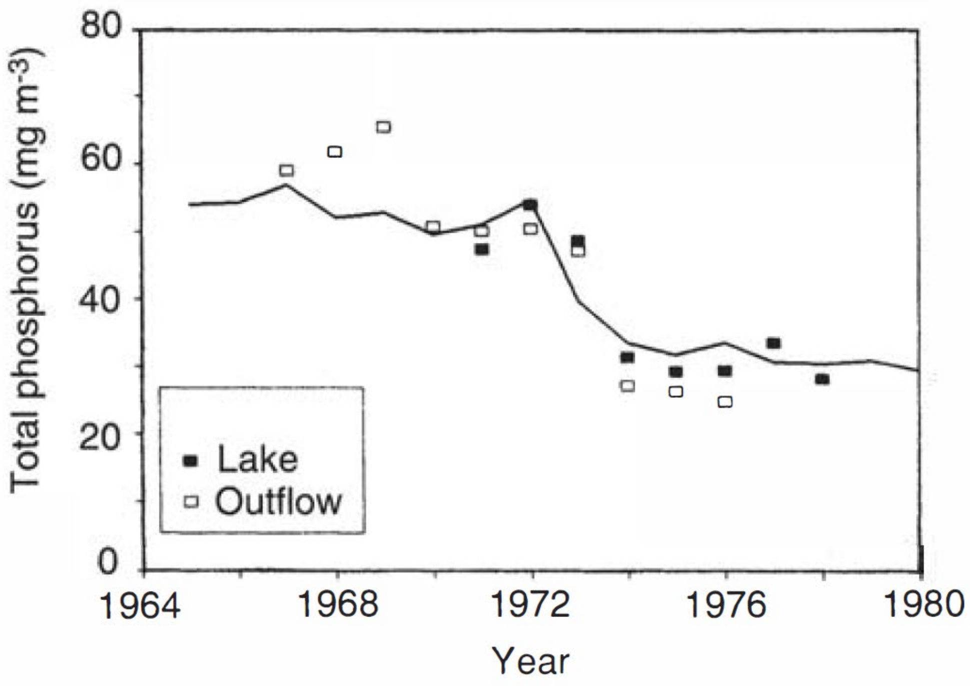


FIGURE 29.12
Recent total phosphorus concentration for Shagawa Lake as simulated by the phosphorus-oxygen model.

The lake simulation between 1964 and 1980 is shown.

Pre-diversion levels are at 50-60 mg m^{-3}. The initial drop of about 25 mg m^{-3} is close to the observed drop. The slow recovery is close to the observed data.

Application: Shagawa Lake

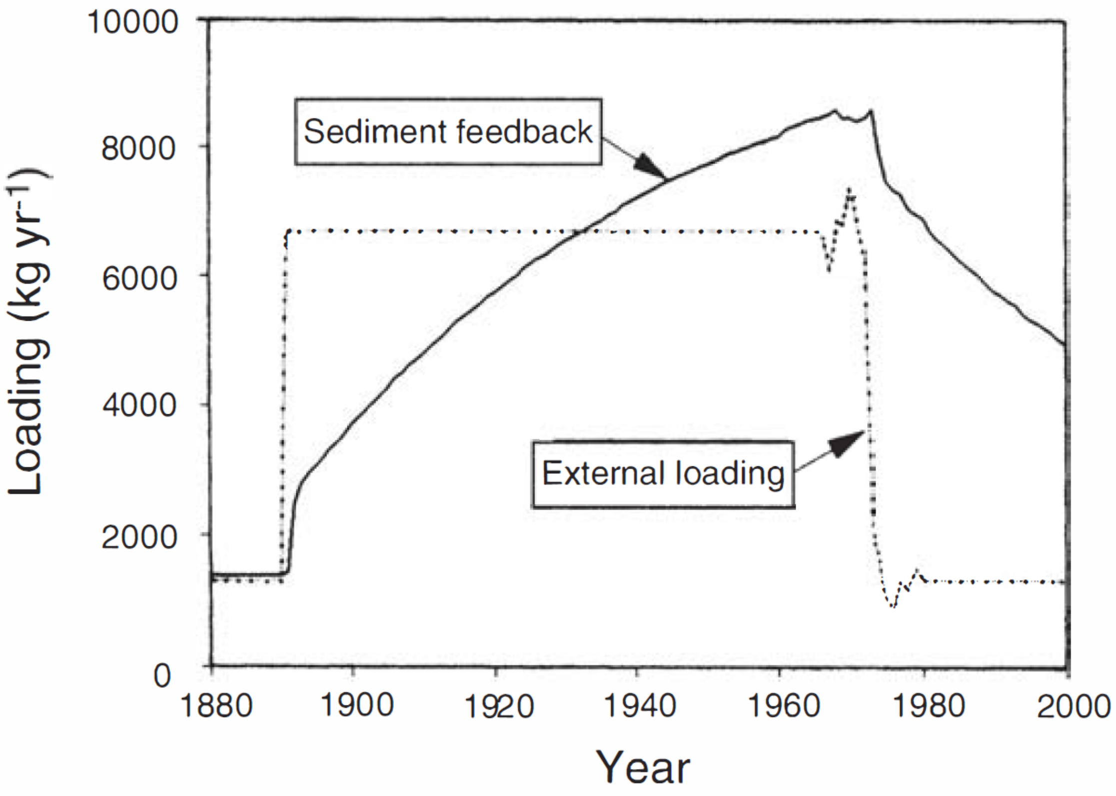
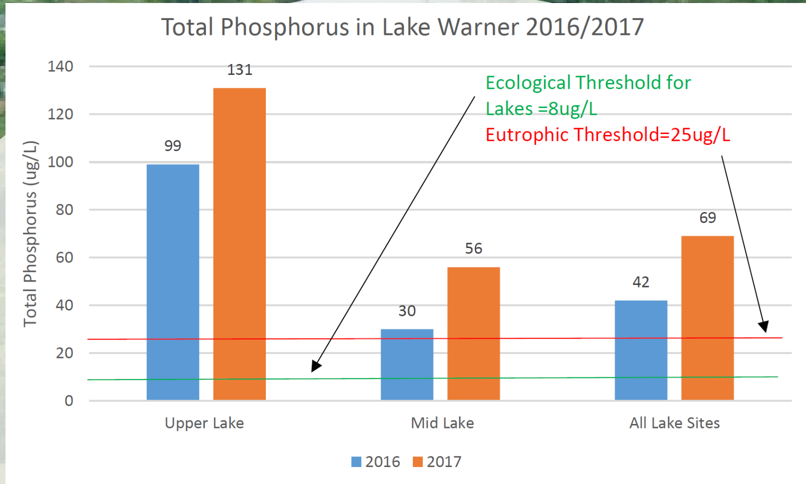


FIGURE 29.13
Long-term trends of sediment recycle (thick line) of phosphorus for Shagawa Lake as simulated by the phosphorus-oxygen model. The external loading is superimposed (thin line) for comparison.

Here we plot external and internal loadings. Internal loading takes many decades to build up. After the advanced treatment there is a steep decrease then gradual decrease.

This is preliminary framework for assessing the impact of sediment feedback of phosphorus on long-term lake recovery.



Depth, Chlorophyll a, Secchi Disk Depth, Areal Hypolimnetic Oxygen Demand, Sediment-Water interaction….

You have 1 million dollars to invest in Lake Warner; how, when, where do you act? Why?

Simplest Seasonal Approach

Temperature has a profound effect on mass cycling within the water column. The surface layer is the epilimnion (warm and well-illuminated). The bottom layer is the hypolimnion (reducing turbulent vertical mixing), where settling and diffusion transfer across thermocline.

This cycle is described here.

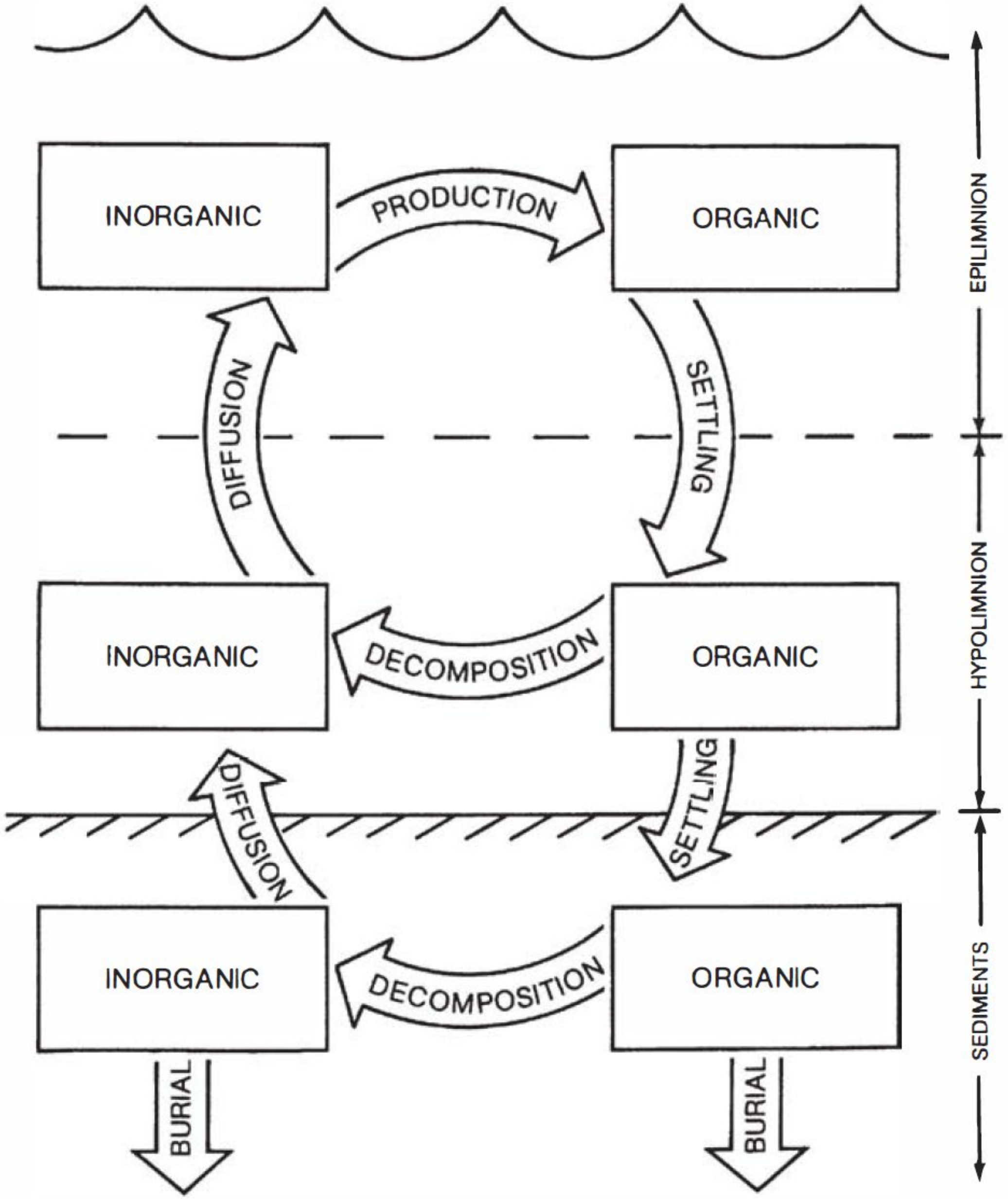


FIGURE 29.14
Idealized representation of the cycle of production and decomposition that plays a critical role in determining the vertical distribution of matter in stratified lakes.

Simplest Seasonal Approach

Many constituents occur in various chemical and/or biological forms that are subject to transformation due to thermal stratification of water column.

Simons and Lam (1980) defined this as the simplest seasonal approach.

1. Phosphorus is separated into two components: Soluble Reactive Phosphorus (SRP) and Phosphorus that is not soluble reactive phosphorus (NSRP).

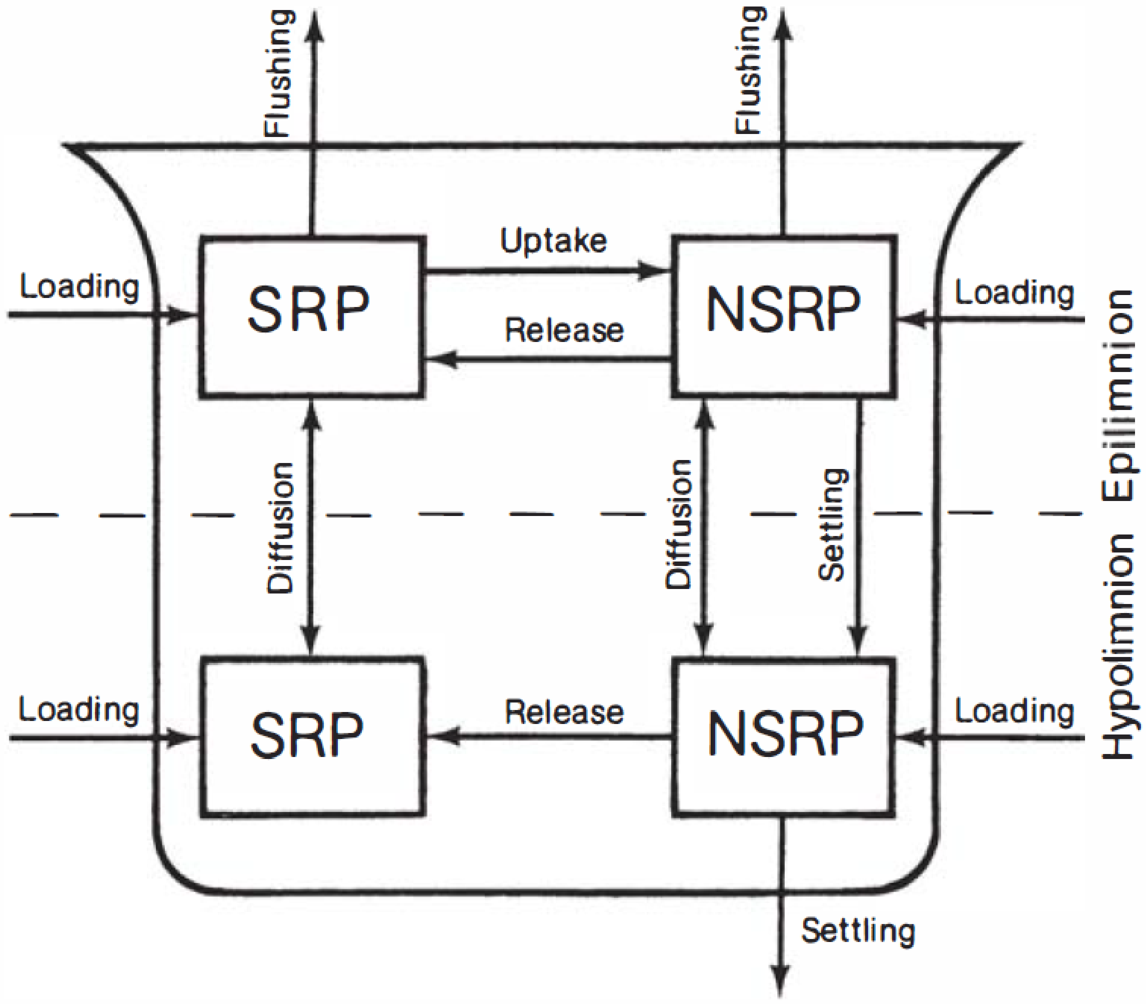


FIGURE 29.15
Schematic representation of the “simplest seasonal approach” developed by O’Melia, Imboden, and Snodgrass.

Simplest Seasonal Approach

2. Lake is segmented spatially into well-mixed upper and lower layers of constant thickness

3. The year is divided into two seasons: summer (stratified) and winter (well-mixed).

4. Linear first-order differential equations are used to characterize the transport kinetics representing the mass exchange between components.

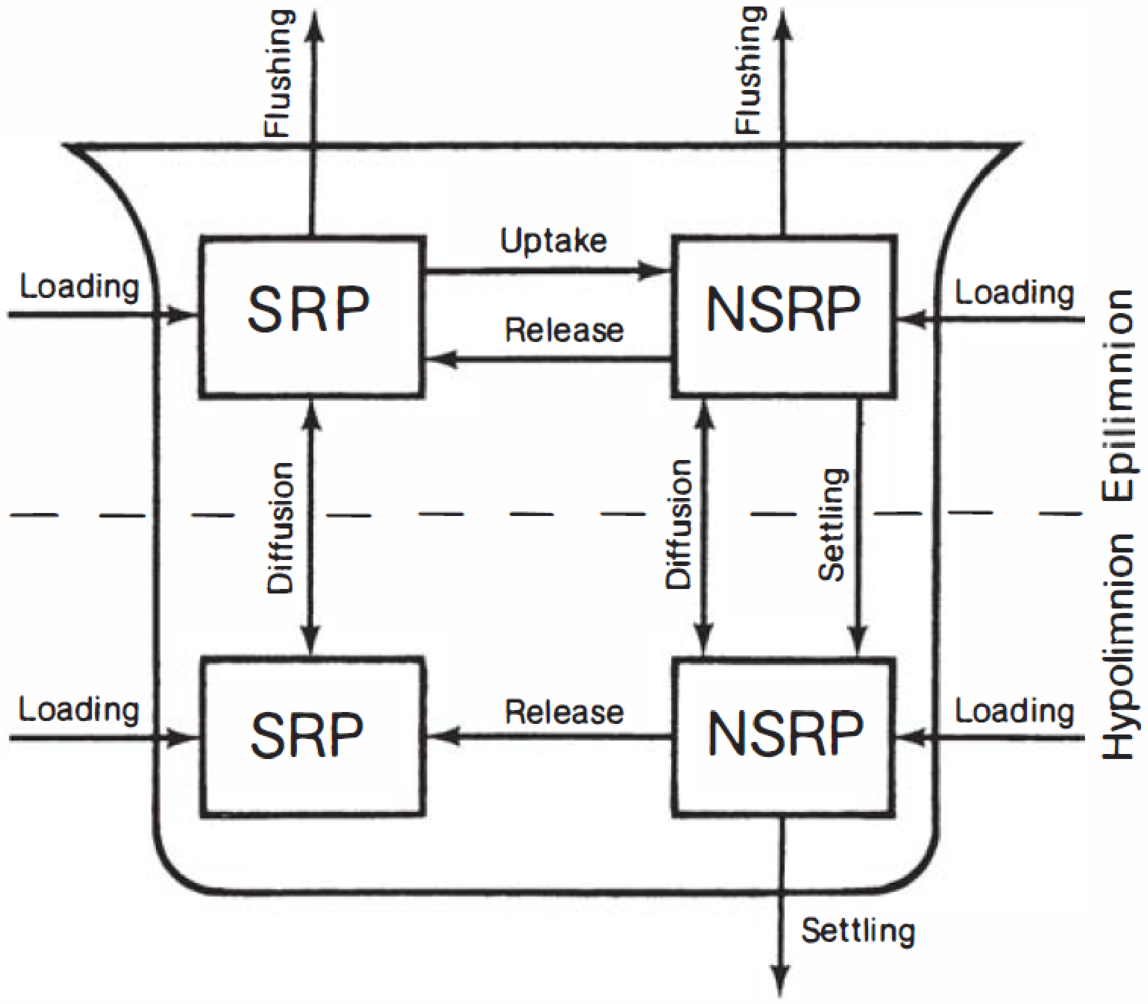


FIGURE 29.15
Schematic representation of the "simplest seasonal approach" developed by O'Melia, Imboden, and Snodgrass.

Simplest Seasonal Approach

-Waste inputs: Loading $(W_{s,e})$

-Transport: Flushing $(Qp_{s,e})$;
$v_t A_t (p_{s,h} - p_{s,e})$

-Settling: $v_e A_t p_{n,e}$

-Uptake: $k_{u,e} V_e p_{s,e}$

-Release: $k_{r,e} V_e p_{n,e}$

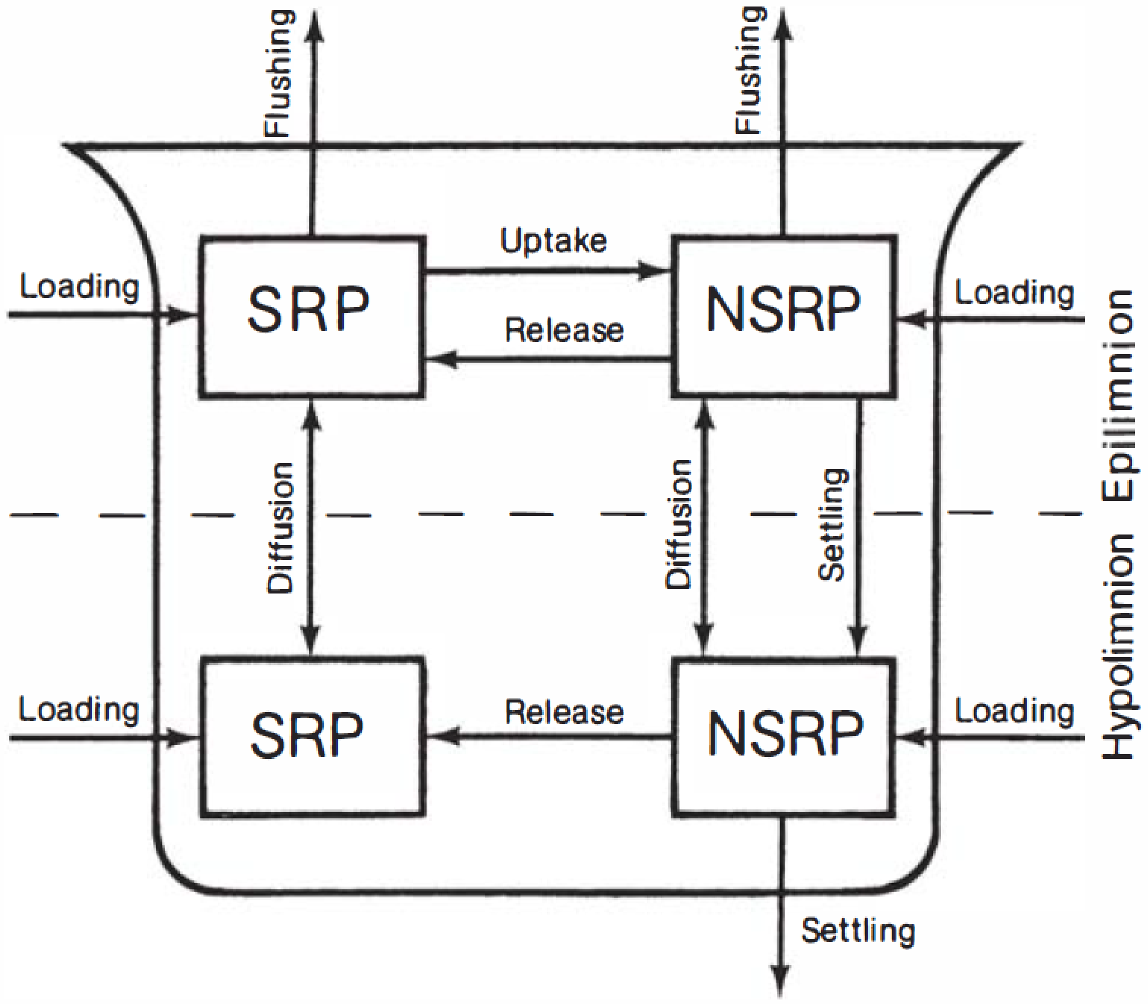


FIGURE 29.15
Schematic representation of the "simplest seasonal approach" developed by O'Melia, Imboden, and Snodgrass.

Simplest Seasonal Approach

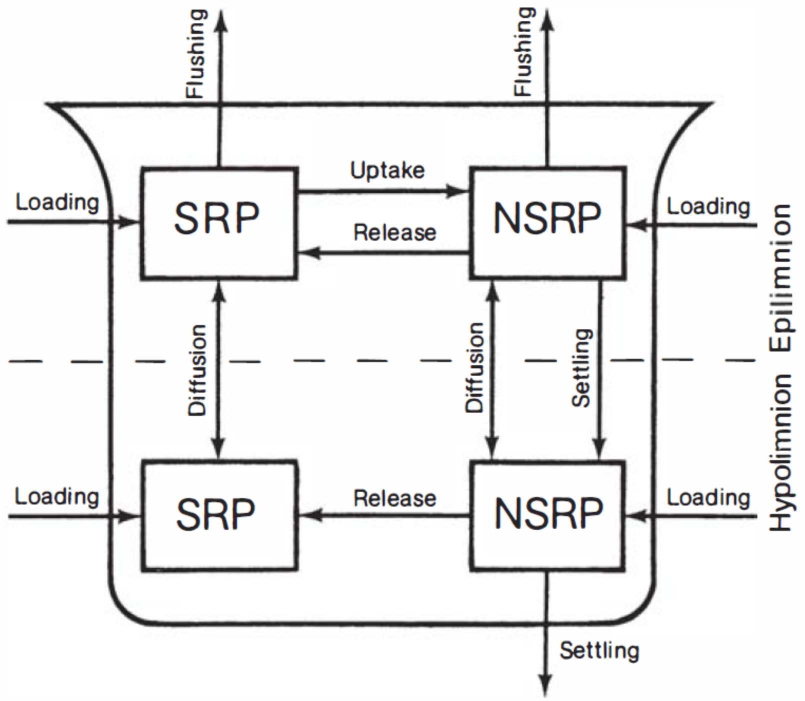


FIGURE 29.15
Schematic representation of the "simplest seasonal approach" developed by O'Melia, Imboden, and Snodgrass.

The loss from NSRP represents a gain for the SRP pool.

In the epilimnion:

$$V_e \frac{dp_{s,e}}{dt} = W_{s,e} - Qp_{s,e} + v_t A_t (p_{s,h} - p_{s,e}) - k_{u,e} V_e p_{s,e} + k_{r,e} V_e p_{n,e}$$

$$Accum = Load - Flush + Diffusion - Uptake + Release$$

Simplest Seasonal Approach

TABLE 29.4
Typical ranges of parameters for the simplest seasonal approach for modeling phosphorus

Parameter	Season	Symbol	Range[†]	Units
Epilimnetic uptake rate	Summer	$k_{u,e}$	0.1–5.0	d^{-1}
	Winter	$k_{u,e}$	0.01–0.5	d^{-1}
Epilimnetic release rate	Summer	$k_{r,e}$	0.01–0.1	d^{-1}
	Winter	$k_{r,e}$	0.003–0.07	d^{-1}
Hypolimnetic release rate	Summer	$k_{r,h}$	0.003–0.07	d^{-1}
	Winter	$k_{r,h}$	0.003–0.07	d^{-1}
Settling velocity	Annual	v_e, v_h	0.05–0.6	$m\ d^{-1}$

[†]Values are taken primarily from Imboden (1974) and Snodgrass (1974).

Mass balances for the remaining pools are:

$$V_e \frac{dp_{n,e}}{dt} = W_{n,e} - Qp_{n,e} + v_t A_t (p_{n,h} - p_{n,e}) + k_{u,e} V_e p_{s,e} - k_{r,e} V_e p_{n,e} - v_e A_t p_{n,e}$$

$$V_h \frac{dp_{s,h}}{dt} = W_{s,h} + v_t A_t (p_{s,e} - p_{s,h}) + k_{r,h} V_h p_{n,h}$$

$$V_h \frac{dp_{n,h}}{dt} = W_{n,h} + v_t A_t (p_{n,e} - p_{n,h}) - k_{r,h} V_h p_{n,h} + v_e A_t p_{n,e} - v_h A_t p_{n,h}$$

Application to Lake Ontario

Parameters for Lake Ontario are summarized in the table.

The heat exchange coefficient across the thermocline is 0.0744 m d^{-1} during the summer.

TABLE 29.6
Kinetic parameters for Lake Ontario in the early 1970s

Parameter	Season	Value	Units
$k_{u,e}$	Summer	0.36	d^{-1}
	Winter	0.045	d^{-1}
$k_{r,e}$	Summer	0.068	d^{-1}
	Winter	0.005	d^{-1}
$k_{r,h}$	Summer	0.005	d^{-1}
	Winter	0.005	d^{-1}
v_e, v_h	Annual	0.103	m d^{-1}

TABLE 29.5
Information on Lake Ontario in the early 1970s

Parameter	Symbol	Value	Units
Area			
Surface	A_s	19,000	10^6 m^2
Thermocline	A_t	18,500	10^6 m^2
Mean depth			
Whole lake	H	86	m
Epilimnion	H_e	15	m
Hypolimnion	H_h	71	m
Volume			
Whole lake	V	1634	10^9 m^3
Epilimnion	V_e	254	10^9 m^3
Hypolimnion	V_h	1380	10^9 m^3
Outflow	Q	212	10^9 m^3 yr^{-1}
SRP load			
Epilimnion	$W_{s,e}$	4000	10^9 mg yr^{-1}
Hypolimnion	$W_{s,h}$	0	10^9 mg yr^{-1}
NSRP load			
Epilimnion	$W_{n,e}$	8000	10^9 mg yr^{-1}
Hypolimnion	$W_{n,h}$	0	10^9 mg yr^{-1}

Application to Lake Ontario

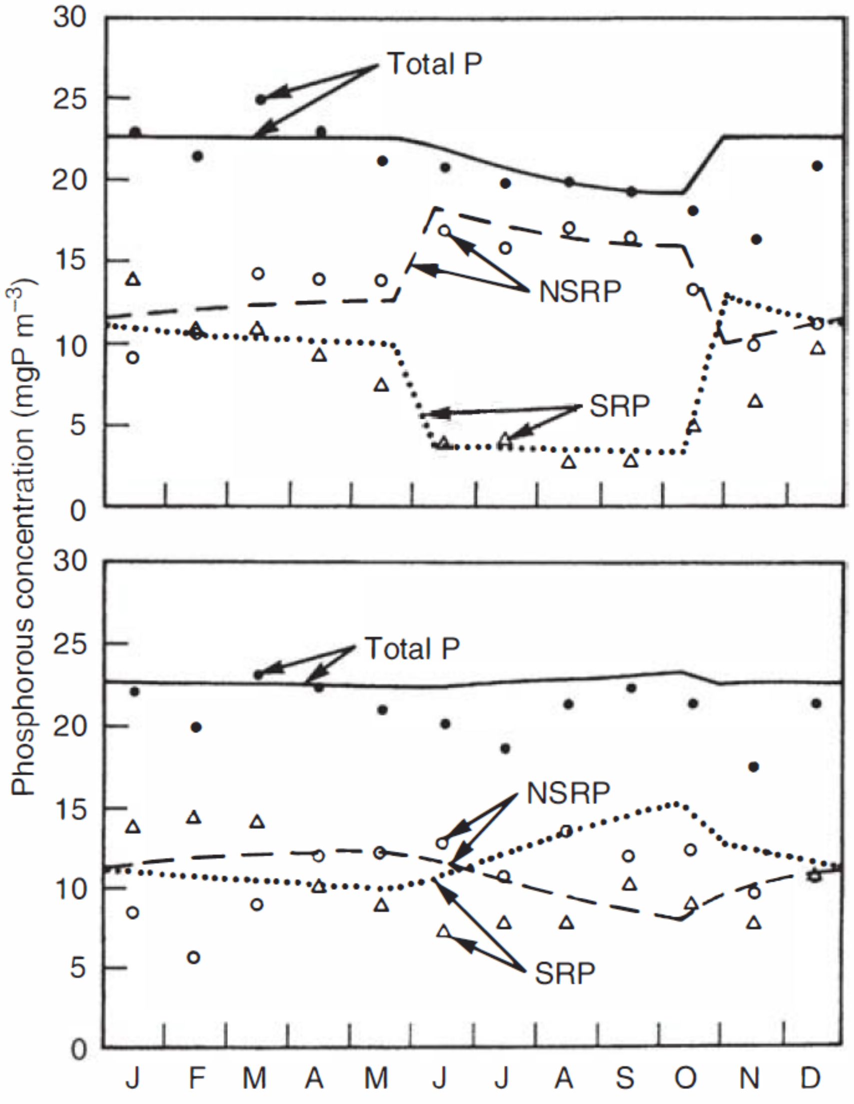


FIGURE 29.16
Data and simulation results using the simplest seasonal approach for total phosphorus in Lake Ontario. (Top—epilimnion; Bottom—hypolimnion).

A fourth-order Runge-Kutta technique is used to solve the phosphorus pools.

The epilimnion shifts from SRP to the NSRP fraction during the summer due to large uptake rate.

The hypolimnion is more stable throughout the year.

Kinetic Segmentation

There are three basic rationales underlying kinetic segmentation.

First, division of matter can be based on measurement techniques, e.g. SRP/NSRP scheme.

Second, segmentation can have a mechanistic basis. (kinetic characteristics, e.g. settling)

Finally, segmentation scheme can have a management basis. (phytoplankton pool for eutrophication)

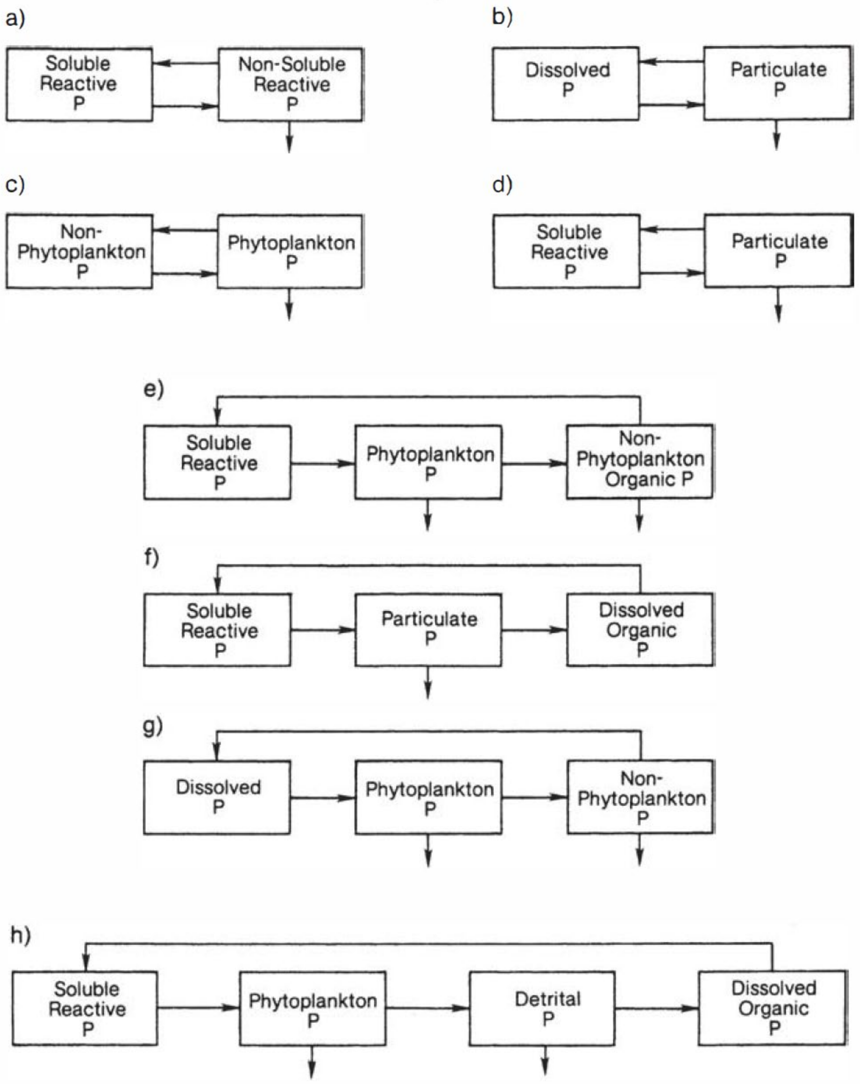


FIGURE 29.17
Alternative kinetic segmentation schemes to model seasonal phosphorus dynamics.