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Elementary Fluid Mechanics CEE 357-02
Fall 2019- December 06
Exam 3

Circle the correct answer or fill in the blank

1. (**3pt**) Calculation of the wetted perimeter (P) requires summation of_____ inside a rectangular channel for which water flows through a fixed cross section:

(a) all of the sides, bottom, and free surface
(b) the sides and the bottom, but not the free surface
(c) only the portion of the sides in contact with water, and the bottom

2. (**3pt**) Surface wave speed can be used to calculate the
(a) Prandtl number
(b) Mach number
(c) Froude number

3. (**3pt**) The classification scheme that allows for the distinction between gradually varying flow and rapidly varying flow requires that we assume_____ for gradually varying flow and _____ for rapidly varying flow (phrase).

One-dimensional approximation, multidimensional flow

Solve and show your work.

4. (**18 pts**) Water flows down a rectangular channel that is 4 ft wide and 2 ft deep. The flow rate is 28,000 gal/min. Estimate the Froude number of the flow. Round the answer to two decimal places.

Using BG units, convert 28,000 gallons/min to ft^3/s.

$$28{,}000\ \frac{gal}{min} \times \frac{1\ ft^3/s}{448.8\ gal/min} = 62.38\ ft^3/s$$

The velocity and the wave speed are:

$$V = \frac{62.38\ ft^3/s}{(4\ ft)(2\ ft)} = 7.80\ ft/s \qquad (6\ pts)$$

$$c_0 = \sqrt{gy} = \sqrt{(32.2) \times (2ft)} = 8.02\ \frac{ft}{s} \qquad (6\ pts)$$

$$Froude\ number = Fr = \frac{V}{c_0} = \frac{7.80\ ft/s}{8.02\ \frac{ft}{s}} \approx 0.97 \qquad (6pts)$$

Name_____

Circle the correct answer

5. (**3 pt**) An elementary wave traveling on the surface of a fluid within a supercritical regime would _____relative to the flow of the fluid.

(a) flow downstream
(b) remain stationary
(c) flow upstream

6. (**3 pt**) The best of all possible channel cross-sections (maximizing R_h, minimizing P) is _____.

(a) a square with the relation b = 2y
(b) a trapezoid
(c) a semi-circle

7. (**3 pt**) Subcritical flow requires a height (y) above the channel bottom that is _____.

(a) less than the critical depth (y_c)
(b) greater than the critical depth (y_c)
(c) equal to the critical depth (y_c)

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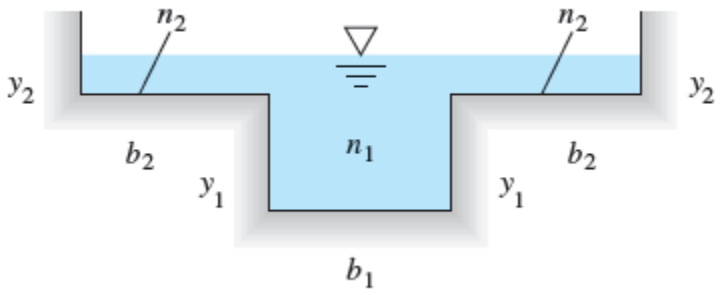
Solve and show your work.

8. Uniform flow in a Channel. (**28 pts**)

In flood stage, a natural channel often consists of a deep main channel plus two floodplains, as shown in the image below. The floodplains are often shallow and rough. If the channel has the same slope everywhere, how would you analyze this situation for the discharge? Suppose that y_1 = 20 ft, y_2 = 5 ft, b_1 = 50 ft, b_2 = 100 ft, n_1 = 0.020, n_2 = 0.040, and with a slope of 0.0002. Estimate the discharge in ft^3/s.

Round the final answer to the nearest whole number.



$$Deep\ channel: \quad Q_1 = \frac{1.49}{0.020}(25 \times 50)\left(\frac{(25 \times 50)}{20 + 50 + 20}\right)^{2/3}(0.0002)^{1/2} \approx 7609.56$$

$(8\ pts)$

$$Flood\ plains: \quad Q_2 = \frac{1.49}{0.040}(5 \times 100)\left(\frac{(5 \times 100)}{5 + 100 + 0}\right)^{2/3}(0.0002)^{1/2} \approx 745.53$$

$$2\,Q_2 = 2 \times (745.53) = 1491.06 \qquad (10pts)$$

$$Q_{total} = Q_1 + 2Q_2 = 7609.56 + 745.53 = 9{,}100 \pm 2\%\ ft^3/s \qquad (10pts)$$

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Circle the correct answer

9. (**2 pt**) A supercritical flow regime will_____ when flowing over a depression:

(a) stay the same depth
(b) decrease in depth
(c) increase in depth

10. (2 **pt**) A subcritical flow at height (y_1) that encounters a sluice gate and smoothly accelerates below the gate will contract to a new height (y_2) that is about _____ less than the gate height (H).

(a) 60 percent.
(b) 40 percent
(c) 2/3^{rds}

11. (2 **pt**) A steady, stable hydraulic jump has this range of Froude numbers: _____.

(a) Fr =1.7 to 2.5

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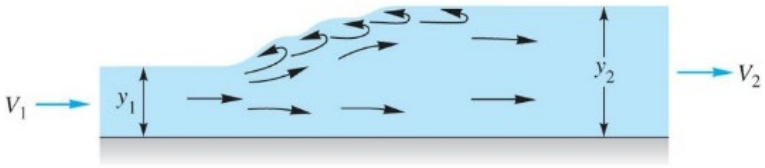
(b) Fr = 4.5 to 9.0
(c) Fr > 9.0

Solve and show your work.

12. Hydraulic Jump **(30 Pts)**

Water flowing in a wide channel 25 cm deep suddenly jumps to a depth of 2.9 m.

(a) Estimate the downstream Froude number (Fr_2), Round the final answer to three decimal places.
(b) Estimate the flow rate per unit width (q_1), Round the final answer to two decimal places.
(c) Estimate the critical depth (y_c), Round the final answer to two decimal places.
(d) Estimate the percentage of dissipation (%). Round the final answer to two decimal places.



$$\frac{2y_2}{y_1} = -1 + (1 + 8Fr_1^2)^{1/2}$$

$$\frac{2(1.5\ m)}{(0.25m)} = -1 + (1 + 8Fr_1^2)^{1/2}$$

$$Fr_1^2 = \left[\left(\frac{2(1.5\ m)}{(0.25m)} + 1\right)^2 - 1\right]/8$$

$$Fr_1 = 4.58$$

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} \rightarrow \quad V_1 = Fr_1\sqrt{gy_1} = 4.58\sqrt{9.81\ (0.25m)} = 7.177$$

$$V_1 = \frac{V_1 y_1}{y_2} = \frac{(7.177)(0.25m)}{1.5m} = 1.196\ m/s$$

$$(a) \quad Fr_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{1.196\frac{m}{s}}{\sqrt{9.81\ (1.5m)}} = 0.312 \qquad (9pts)$$

$$(b) \quad q = V_1 y_1 = \left(1.196\frac{m}{s}\right)(0.25\ m) = 1.79\ m^2/s \qquad (7pts)$$

$$(c) \quad y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(1.79)^2}{9.81}\right)^{1/3} = 0.69\ m \qquad (7pts)$$

$$(d) \quad E_1 = y_1 + \frac{V_1^2}{2g} = \quad 2.875\ m; \qquad h_f = \frac{(y_2 - y_1)^3}{4y_1 y_2} = \frac{(1.5 - 0.25)^3}{4 * (0.25)(1.5)}\ 1.30\ m$$

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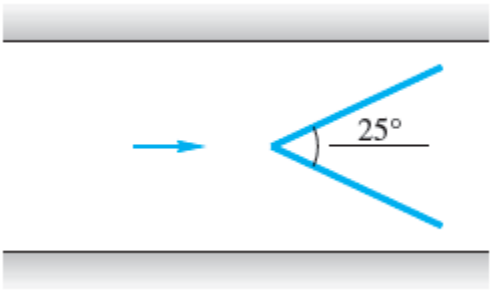
$$\%\ dissipation = \frac{h_f}{E_1} = \frac{1.3}{2.875} = 45.29\ \% \qquad (7pts)$$

----------BONUS----------

Bonus: A pencil point piercing the surface of a rectangular channel flow creates a 25° half-angle wedge like wave, as shown in the image below. The channel surface is painted steel and the depth is 65 cm.



(2pts) Determine the Froude number. Round the final answer to two decimal places

Using the relation for the angle of the waves:

$$\mu = \sin^{-1}\left(\frac{1}{Fr}\right) \ or \ \ Fr = \frac{1}{\sin\mu} \ ; \ \ Fr = \csc\mu = \csc(25°) = 2.37$$

(2pts) Determine the critical depth. Round the final answer to two decimal places.

$$V = Fr \times V_c = 2.37\sqrt{(9.81)(0.65)} = 5.98\ ft/s$$

$$Flow\ rate \qquad q = Vy = 5.98\ (0.65) = 3.88\frac{m^2}{s}$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(3.88)^2}{9.81\ m/s^2}\right)^{1/3} \approx 1.15\ m$$

Bonus (2 points): A cast iron duct (n =0.013) of diameter 2.9 m is flowing half-full at 33 m^3/s. Determine the slope of this duct if the flow is uniform. Round answer to four decimal places.

$$Q = \frac{1.0}{n} A R_h^{2/3} S_o^{1/2}$$

$$P = \pi R = \pi(1.45m)$$

$$R_h = \frac{A}{P} = \frac{\pi R^2/2}{\pi R} = 0.725$$

$$S_o^{1/2} = \frac{Qn}{AR_h^{2/3}} = \frac{(33\ m^3/s)(0.013)}{(\frac{\pi(1.45)^2}{2})(0.725^{2/3})} = 0.16096$$

$$S_o = 0.16096^2 = 0.02591$$

Bonus (2 points): Find the flow rate (in cubic ft per second) of a half-full glass circular pipe with a slope of 2° and diameter 4.5 ft.

$$Q = \frac{1.49}{n} A R_h^{2/3} S_o^{1/2}$$

$$P = \pi R = \pi(2.25m)$$

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$$R_h = \frac{A}{P} = \frac{\pi R^2/2}{\pi R} = \frac{2.25}{2} = 1.125$$

$$Q = \frac{1.49}{0.010}(\pi(2.25)^2/2)(1.125)^{2/3}(\tan 2°)^{1/2} = 239.51\ ft^3/s$$